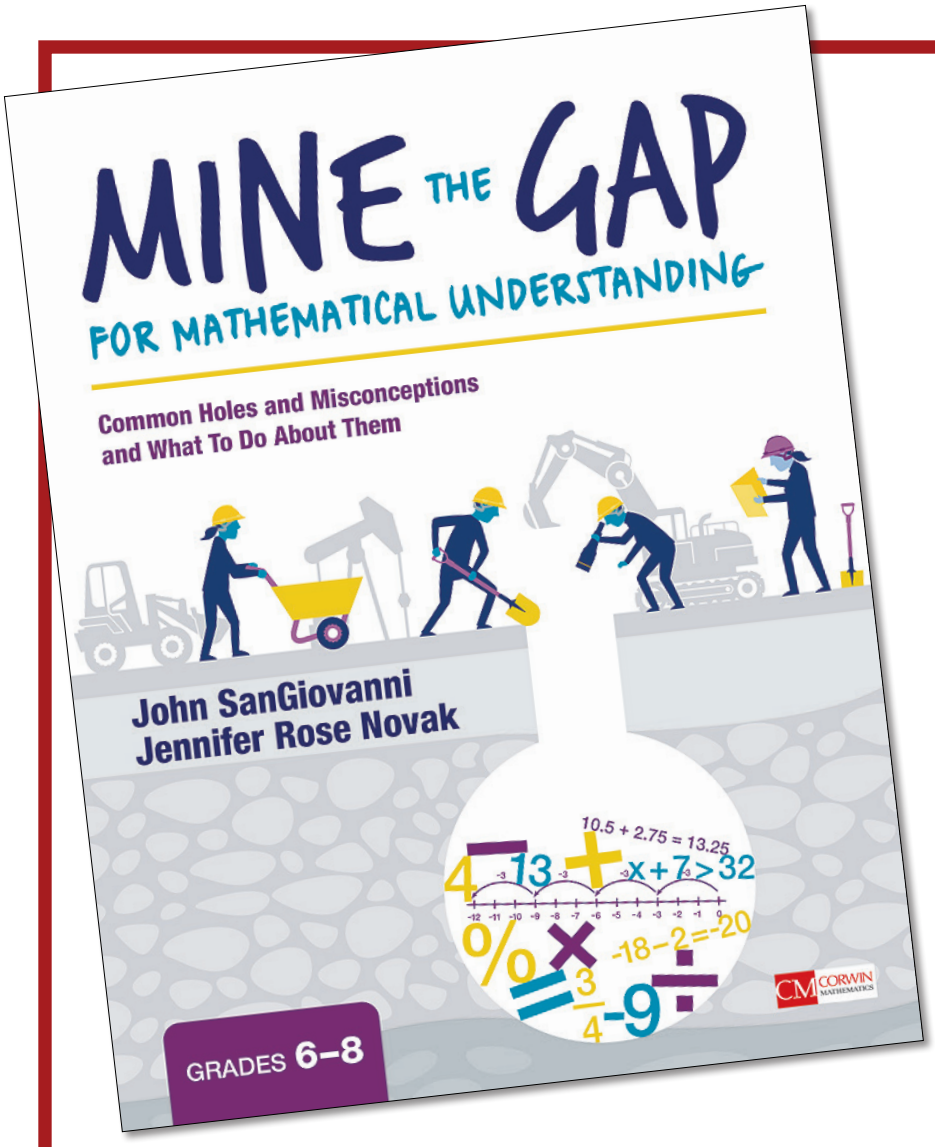


Thank you

FOR YOUR INTEREST IN CORWIN

Please enjoy this complimentary excerpt from *Mine the Gap for Mathematical Understanding, Grades 6-8*. In this task, students are provided with a true equation and asked to create three new equations based on a stated condition.

LEARN MORE about this title, including Features, Table of Contents, and Reviews.

BIG IDEA 13

BIG IDEA 13
Addition With Integers

TASK 13A

Jerry added −27 + 13 and got −14. He wrote −27 + 13 = −14.

The next three times he added, his sum was less than −14.

Write the numbers he might have added.

Use pictures, numbers, or words to explain your thinking for one of the equations above.

About the Task

Reasoning about addition with integers is critical for understanding the results of calculations. This impacts our students' ability to consider the reasonableness of their sums. Considering what happens when we combine positive and negative integers and the relationship between addends aids the development of computational fluency with integers. *In this task, students are provided with a true equation and asked to create three new equations based on a stated condition.* As with other tasks, discussing the wide range of results with the class can help develop generalizations and deeper understanding of the mathematics.

MODIFYING THE TASK

This task can be easily modified to feature different integers or operations.

Anticipating Student Responses

The task is intended to determine if students can make sense of the relationship between addends as well as positive and negative numbers. Students might adjust one of the addends in each equation to be less than one of the original

PAUSE AND REFLECT

- **How does this task compare to tasks I've used?**
- **What might my students do in this task?**

Visit this book's companion website at **resources.corwin.com/minethegap/6-8** for complete, downloadable versions of all tasks.

addends. Others might adjust both addends. Some students might adjust the addends correctly but find an incorrect sum, though the sum is less than the original. Others might adjust the addends incorrectly and find an incorrect sum that does satisfy the prompt. Some students might fully show a calculation to prove their new sum and how it compares to the original. Students with more advanced reasoning may simply explain how their adjusted addend(s) must result in a lesser sum.

NOTES

WHAT THEY DID

Student 1

Student 1 replicates the digits in the prompt. He moves the negative sign to different locations in each line. His work may show a misconception related to fact families. It may also be connected to thoughts about multiplication and division with integers. His explanation below may be evidence of the latter.

Student 2

Each of Student 2's expressions is accurate in relationship to the prompt. He justifies one of his sums by showing how he would add on a number line. Interestingly, Student 2 begins to approach the intent of the task by unchanging the first addend. Yet, he doesn't quite establish that the sum of −27 and another number will be less than −27 and 13 when that number is less than 13.

USING EVIDENCE

What would we want to ask these students? What might we do next?

Student 1

This task is intended to shed light on students' reasoning about how sums are influenced by changing addends. However, Student 1 uncovers a more significant misunderstanding. He is unable to reason about changing sums because he fundamentally misunderstands addition with integers. He may be a good example of what happens when students are prematurely taught rules in mathematics without deeper conceptual understanding. It is clear that we need to restart our work with addition of integers with Student 1. We want to be sure that in doing so we fully disconnect any ideas about the "rules" for adding integers. Instead, we should shift our focus to what is happening and why it is happening when integers are added. We should also be sure to use contexts and varied representations to develop this understanding.

Student 2

Student 2 shows understanding that we can build on. He is able to find sums of integers. Our next work is to help him begin to reason about the results of his computations. We can do this in all sorts of ways. One approach is to consistently estimate or discuss what a sum or solution might be before finding the actual. After finding the actual, we can then compare results. This type of approach helps our students develop reasonableness of their answers.

TASK 13A: Jerry added −27 + 13 and got −14. He wrote −27 + 13 = −14. The next three times he added, his sum was less than −14. Write the numbers he might have added. Use pictures, numbers, or words to explain your thinking for one of the equations you wrote.

Student Work 1

Jerry added -27 + 13 and got -14. He wrote -27 + 13 = -14.

The next three times he added his sum was less than -14.

Write the numbers he might have added.

-27 + -13

27 + 13

27 + -13

Use pictures, numbers, or words to explain your thinking for **one** of the equations above.

Negative Plus negad = Negative

Student Work 2

Jerry added -27 + 13 and got -14. He wrote -27 + 13 = -14.

The next three times he added his sum was less than -14.

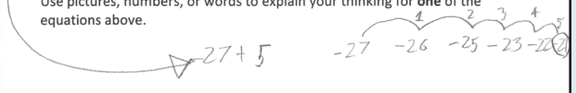
Write the numbers he might have added.

-27 + 12

-27 + 5

-27 + 0

Use pictures, numbers, or words to explain your thinking for **one** of the equations above.



WHAT THEY DID

Student 3

Like Student 2, Student 3 does not change the first addend in each of his expressions. We can leverage this to help him reason about how changing the second addend will affect the sum. His writing relates how adding a positive number to a negative number will impact the sum. Essentially, he has the ingredients for advancing his reasoning skills.

Student 4

Student 4 shows quite a generalization for creating expressions that will satisfy the prompt. In two of the three expressions, he adds two negative numbers, each containing a negative that is quite less than the sum of the original equation. His third expression does add a negative and a positive, but he compensates by creating a considerably less negative number while leaving the second addend unchanged.

USING EVIDENCE

What would we want to ask these students? What might we do next?

Student 3

Our recording of expressions and equations must be intentional so that students can see patterns and relationships between them. Many of us were not taught to record mathematics this way on chalk or white board.

As noted, Student 3 has ingredients for reasoning about sums of negative and positive numbers. We might approach his development similar to Student 2. They might both benefit from discussion with one another. We might also be sure to work with situations in which similar equations are recorded in a fashion that helps them see relationships. *For example, we might record –27 + 13 = –14, –27 + 12 = –15, and –27 + 11 = –16 on top of one another.* We can then ask students to describe what remains the same and what changes with each equation. We can ask them to discuss the magnitude of each change. We might also ask them to predict what the next equation might be. Examining these sorts of patterns within computations may be new to our students and so we should remain patient and resilient by sticking with the activity for several days.

Student 4

Student 4 shows a different type of reasoning that is useful when reasoning about computation. However, it may not be intentional, so it is up to us to investigate further. Essentially, he shows in his second expression that two addends will generate a smaller sum if they are both less than two other addends. His third prompt builds on the idea of unchanged addends. Like the others, Student 4 shows ideas about numbers that need nurturing. We should be sure to offer consistent opportunities to play with expressions or equations and to discuss our results so that students develop the type of reasoning that goes well beyond mechanical computation.

TASK 13A: Jerry added −27 + 13 and got −14. He wrote −27 + 13 = −14. The next three times he added, his sum was less than −14. Write the numbers he might have added. Use pictures, numbers, or words to explain your thinking for one of the equations you wrote.

Student Work 3

Jerry added -27 + 13 and got -14. He wrote -27 + 13 = -14.

The next three times he added his sum was less than -14.

Write the numbers he might have added.

-27 + 5

-27 + 10

-27 + 8

Use pictures, numbers, or words to explain your thinking for **one** of the equations above.

-27+5=-22because if you add a positive number to a negative number it gets the negative number closer to zero.

Student Work 4

Jerry added -27 + 13 and got -14. He wrote -27 + 13 = -14.

The next three times he added his sum was less than -14.

Write the numbers he might have added.

-57 + -1

-176 + -35

-296 + 13

Use pictures, numbers, or words to explain your thinking for **one** of the equations above.

I know sence -296 is much lower then -27 the answer should be lower sence your adding the same number.

OTHER TASKS

- **What will count as evidence of understanding?**
- **What misconceptions might you find?**
- **What will you do or how will you respond?**

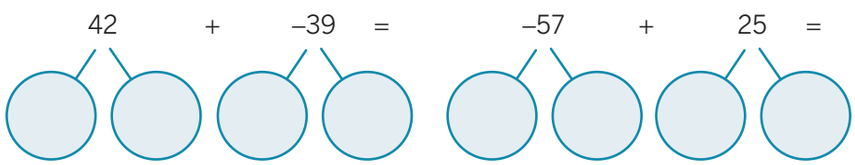
Visit this book's companion website at **resources.corwin.com/minethegap/6-8** for complete, downloadable versions of all tasks.

TASK 13B: Add each. Use a number line to show how you added.

35 + 45 **−37 + −36** **28 + −42** **−15 + 61**

Task 13B provides insight into how students combine integers. It helps us see if they decompose integer addends in ways similar to whole numbers. The first prompt is intentional as it provides "baseline" information into their approaches to adding in general. Some may add 35 + 40 + 5. Others may add 45 + 30 + 5, while others may think of 30 + 40 + 10. There are a host of ways to manipulate the various expressions. Students may decompose by place value or use friendlies. Students can use either endpoint as a starting point for the computation or work within endpoints of a specific range (e.g., 0 and 100 for 35 + 45). We should look for students who move to the right in each of the four expressions. We should also look for students who add the expressions procedurally to find the sums and then draw on the number line to match the equations.

TASK 13C: Break apart one or both numbers to make them easier to add. Write the sum.



MODIFYING THE TASK

This task features a positive and negative integer addend. It can be modified to feature two positive or two negative integers. In time, we may consider adding a third addend.

As with the other tasks, task 13C is intended to promote number sense and computational fluency. It makes use of number bonds to represent how an integer addend might be decomposed to add to another integer. We want to look for students to decompose in rational ways. In the first prompt, they may think of 42 as 39 and 3. In doing so, they can add −39 + 39 + 3 making the computation considerably easier. In the second, students might think of −57 as −25 and −32. The addition then becomes −25 + 25 + −32. Of course, there are all sorts of ways to decompose integers and other numbers. In this task, we want to look to see if our students decompose to improve the efficiency of calculation.

TASK 13D: **Use the integer chart to add −38 + 41, −29 + 27, −12 + −22, and −32 + 59.**

–40	–39	–38	–37	–36	–35	–34	–33	–32	–31
–30	–29	–28	–27	–26	–25	–24	–23	–22	–21
–20	–19	–18	–17	–16	–15	–14	–13	–12	–11
–10	–9	–8	–7	–6	–5	–4	–3	–2	–1
0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59

This task makes use of an integer chart introduced earlier in this chapter (task 9C). Adding with integer chips may work well for many of our students, but the practicality of the addends is limited. Number lines may also work well. Adding on an integer chart helps students understand how they can apply whole number addition strategies to integers. Using the chart, we may see them decompose an addend and count on by tens and then ones. In time, we should see them count on by multiples of tens and ones. The integer chart also helps students see how combining positive and negative numbers can be done by working through zero. For example, in the expression −38 + 41, we might decompose the 41 into 38 and 3. Then, we would combine or add −38 and 38 to find 0. 0 and 3 more equals 3. So, −38 + 41 = 3.

NOTES

Learn more about the book this lesson is from: http://bit.ly/2OEOatu