
Mathematics Assessment Probes

To differentiate instruction effectively, teachers need diagnostic assessment strategies to gauge their students' prior knowledge and uncover their misunderstandings. By accurately identifying and addressing areas of difficulties, teachers can help their students avoid becoming frustrated and disenchanted with mathematics and can prevent the perception that "some people just aren't good at math." Diagnostic strategies also support instruction that builds on individual students' existing understandings while addressing their identified difficulties. From infancy and through prekindergarten, children develop a base of skills, concepts, and misconceptions about numbers and mathematics (NRC, 2001, p. 157). Understanding and targeting these specific areas of difficulty enables teachers to perform focused and effective diagnostic assessment. The Mathematics Assessment Probes ("Probes") in this book allow teachers to target specific areas of difficulty as identified in research on student learning.

The Probes typically include a prompt or question and a series of responses designed specifically to elicit prior understandings and commonly held misunderstandings that may or may not be uncovered during an instructional unit. In the example in Figure 1.1, students are asked to choose from a selection of responses as well as write about how they determined their answer choice.

This combination of selected response and further explanation helps to guide teachers in making instructional choices based on the specific needs of students. Since not all Probes follow the same format, we will discuss the varying formats later in this chapter. If you are

Figure 1.1 Example of a Probe



Completing Number Sentences

<p>1.</p> $3 + 3 = \square + 5$ <p>Circle the number that belongs in the box:</p> <p>6 11 1</p>	<p>Write about how you got your answer.</p>
<p>2.</p> $8 + 3 = 7 + \square$ <p>Circle the number that belongs in the box:</p> <p>11 4 18</p>	<p>Write about how you got your answer.</p>

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wondering about what other kinds of Probes are included in this book, take a few moments to review two or three additional Probes from Chapters 2–6 before continuing reading, but we strongly suggest that you return to read the rest of this chapter before beginning to use the Probes with your students.

Are you wondering about the Probes? If you are, we suggest reviewing the following Probes as initial examples:

- Name the Missing Number Interview Probe p. 38
- Is it a Triangle? Probe Sort p. 148
- Are They Equivalent? Probe p. 92

Are you wondering about the Probes? If you are, we suggest reviewing the following Probes as initial examples:

- Name the Missing Number Interview Probe p. 38
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- Are They Equivalent? Probe p. 92

At this point, you may be asking; “What is the difference between Mathematics Assessment Probes and other assessments?” Comprehensive diagnostic assessments for primary grade mathematics such as Key Math3 (Pearson) and assessments from the Northwest Education Association (NWEA) as well as the many state- and district-developed assessments can provide

information important for finding entry points and current levels of understanding within a defined progression of learning for a particular mathematics subdomain such as counting and cardinality. Such assessments will continue to play an important role in schools, as they allow teachers to get a snapshot of student understanding across multiple subdomains, often at intervals throughout the year depending on the structure of the assessment.

How are Probes different? Consider the following vignette:

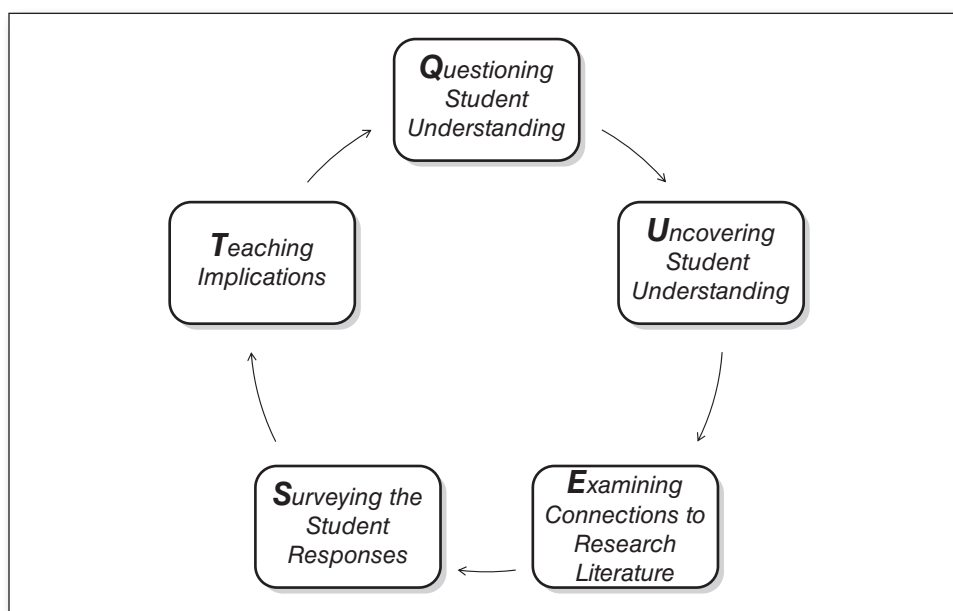
In a primary classroom, students are having a “math talk” to decide which figures are triangles. After using a card sort strategy to individually group picture cards as “triangles” and “not triangles,” the teacher encourages the students to develop a list of characteristics that could be used to decide whether a figure is a triangle. As students share their ideas and come to an agreement, the teacher records the characteristic and draws an example and nonexample to further illustrate the idea. She then gives students an opportunity to regroup their cards, using the defining characteristics they have developed as a class. As the students discuss the results of their sorting process, she listens for and encourages students to use the listed characteristics to justify their choices. Throughout the discussion, the class works together to revise the triangle characteristics already listed and to add additional characteristics that were not included in the initial discussion (excerpt from Keeley & Rose Tobey, 2011, p. 1).

The Probe in this vignette, the *Triangle Card Sort*, serves as a diagnostic assessment at several points during the lesson. The individual elicitation allows the teacher to diagnose students’ current understanding; the conversation about characteristics both builds the teacher’s understanding of what students are thinking and creates a learning experience for students to further develop their understanding of the characteristics of triangles. The individual time allotted for regrouping the cards allows the teacher to assess whether students are able to integrate this new knowledge with former conceptions or whether additional instruction or intervention is necessary.

Rather than addressing a variety of math concepts, Probes focus on a particular subconcept within a larger mathematical idea. By pinpointing one subconcept, the assessment can be embedded at the lesson level to address conceptions and misconceptions while learning is underway, helping to bridge from diagnostic to formative assessment.

Helping all students build understanding in mathematics is an important and challenging goal. Being aware of student difficulties and the sources of those difficulties, and designing instruction to diminish them, are important steps in achieving this goal (Yetkin, 2003). The process of using a Probe to diagnose student understandings and misunderstandings and then responding with instructional decisions based on the new information is the key to helping students build their mathematical knowledge. Let’s take a look at the complete Probe implementation process we call the *QUEST Cycle* (Figure 1.2).

Figure 1.2 Quest Cycle



Source: Adapted from Rose, Minton, & Arline (2007).

- **Questioning student understanding:** Determine the key mathematical understandings you want students to learn.
- **Uncovering student understanding:** Use a Probe to uncover understandings and areas of difficulties.
- **Examining connections to research and educational literature:** Prepare to answer the question: In what ways do your students' understandings relate to those described in the research base?
- **Surveying the student responses:** Analyze student responses to better understand the various levels of understanding demonstrated in their work.
- **Teaching implications:** Consider and follow through with next steps to move student learning forward.

Note that in the *Triangle Sort Vignette*, this cycle is repeated several times within the described instructional period.

The remaining parts of this chapter describe important components of the QUEST Cycle for implementing Probes, including background information on the key mathematics, the structure of the Probes, and connections to the research base. In addition, you will learn about how to get started with administering the Probes.

QUESTIONING STUDENT UNDERSTANDING: DETERMINE THE KEY MATHEMATICAL CONCEPTS YOU WANT STUDENTS TO LEARN

The Common Core State Standards for Mathematics (referred to as the Common Core) define what students should understand and are the basis

for the targeted mathematics concepts addressed by the Probes in this book. These understandings include both conceptual and procedural knowledge, both of which are important for students' mathematical development.

Research has strongly established that proficiency in subjects such as mathematics requires conceptual understanding. When students understand mathematics, they are able to use their knowledge flexibly. They combine factual knowledge, procedural facility, and conceptual understanding in powerful ways. (NCTM, 2000, p. 20)

Think about the experience of following step-by-step driving directions to an unfamiliar destination using the commands of a GPS but never having viewed a road map of the area. Although it may be easy to follow the directions one step at a time, if you lose your satellite reception, you will likely not know where to turn next or even which direction to head. Using a GPS without a road map is like learning procedures in math without understanding the concepts behind those procedures. Learners who

Table 1.1 Procedural Versus Conceptual Understanding

<i>Factual Knowledge: Procedures, Skills, and Facts</i>	<i>Accompanying Conceptual Understanding</i>	<i>Examples</i>
Learn and apply a series of steps	<ul style="list-style-type: none"> • Explain why the steps make sense mathematically • Use reasoning to rebuild the steps if needed • Make connections between alternate steps that also can be used to find the solution 	When adding 23 and 12, can describe and connect two different methods for adding these two-digit numbers Can interpret a graph to tell about a data set
Find the answer	<ul style="list-style-type: none"> • Justify whether the answer makes sense (numerical example: reasoning about the size of numbers and a mathematical operation) • Troubleshoot a mistake • Represent thinking with symbols, models, and/or diagrams • Show flexibility in representing mathematical situations 	Can reason that $13 + 15$ must be between 20 and 30, since there are only two tens plus some ones Can sort a collection of geometric shapes in more than one way by attending to their attributes
Memorize facts	<ul style="list-style-type: none"> • Generate answer quickly when unable to recall a fact (automaticity) 	Has an efficient method to add facts not remembered by recall: $6 + 9$ (add 10; go back 1) $3 + 4$ (doubles plus 1) $3 + 8$ (make a 10 with 2 and 8; add 1 more)

follow the steps of a mathematical procedure, without any conceptual understanding connected to that procedure, may get lost when they make a mistake. Understanding the bigger picture enables learners to reason about a solution and/or reconstruct a procedure.

This relationship between understanding concepts and being proficient with procedures is complex. Table 1.1 provides some examples of each type of understanding for a variety of contexts.

The relationship between understanding concepts and being proficient with procedures is further developed in the examples of the Probes that follow. Both conceptual understanding and procedural flexibility are important goals that complement each other in developing strong mathematical abilities. Each is necessary, and only together do they become sufficient. The examples of Probes in Figures 1.3, 1.4, and 1.5 will further distinguish conceptual and procedural understandings.

Example 1: Chicken and Eggs Probe

In the Chicken and Eggs Probe, students with conceptual and procedural understanding pay attention to the *number* of objects rather than other characteristics, including size and arrangement. The task moves students beyond just counting by asking them to compare “how many” and elicits conceptual understanding of cardinality. The task can also elicit flexibility in determining how to count a set of objects (rote counting versus one-to-one counting versus cardinality). More information about this Probe can be found on pages 29–37.

Example 2: Length of Rope Probe

In the Length of Rope Probe, students with conceptual and procedural understanding pay attention to how the unit (the minicrayon) has been tiled. Students who have conceptual understanding look for repeated tiling of the unit without gaps or overlap and can determine when additional units are needed to determine a length. They understand length measure as more than just where the end of an object aligns to the number of tiled units and that the orientation of the unit matters only when it impacts the unit’s length. More information about this Probe can be found on pages 128–134.

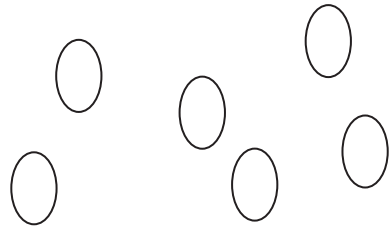
Example 3: Solving Number Stories Probe

In the Solving Number Stories Probe, students with conceptual and procedural understanding pay attention to the context of the problems to determine whether the numbers should be joined, separated, or compared. Rather than focusing solely on key words as a problem-solving approach, these students are able to represent the problem based on an approach that models the situation. Students can solve the problem accurately and can describe how the numbers involved in modeling the problem relate back to the context. More information about this Probe can be found on pages 121–125.

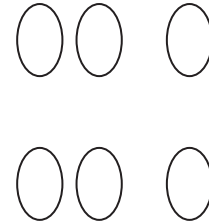


Figure 1.3 Chicken and Eggs Problems 1 and 2

Carla's Eggs 



Bonnie's Eggs 



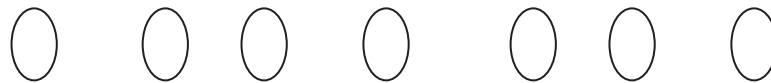
Who has more eggs? Circle the letter.

- A. Carla has more eggs.
- B. Bonnie has more eggs.
- C. Carla and Bonnie have the same number of eggs.

Penny's Eggs 



Dee Dee's Eggs 



Who has more eggs? Circle the letter.

- A. Penny has more eggs.
- B. Dee Dee has more eggs.
- C. Penny and Dee Dee have the same number of eggs.

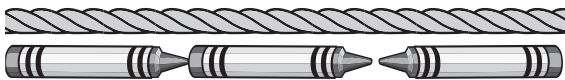
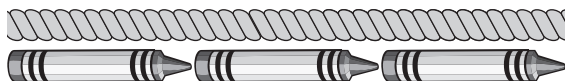
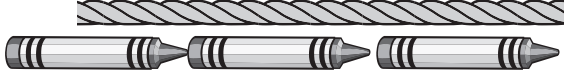


Figure 1.4 Length of Rope

Length of Rope

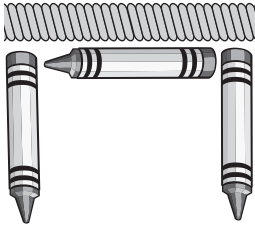
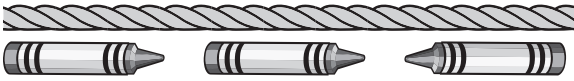
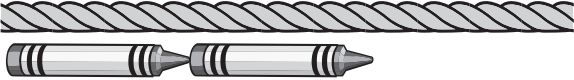
Susie is using minicrayons to measure different-size pieces of rope. The pieces of rope and minicrayons are shown below.

Problems 1-3

Decide if each piece of Susie's rope is 3 minicrayons long.		Circle One
1		Yes No
2		Yes No
3		Yes No

Explain how you decided whether to circle Yes or No:

Problems 4-6

Decide if each piece of Susie's rope is 3 minicrayons long.		Circle One
4		Yes No
5		Yes No
6		Yes No


Explain how you decided whether to circle Yes or No:



Figure 1.5 Solving Number Stories


1. Three students each solved the following problem.
 Mike has 23 toy cars. Susan has 31 toy cars. How many more toy cars does Susan have than Mike?

I think the answer is 54




Lamar

I think the answer is 8



Fran

I don't think the answer is 54 or 8




Tom

Circle the name of the student you agree with. Use words or pictures to show your thinking.


2. Three students each solved the following problem.
 Paula has some grapes. Carlos gave her 18 more grapes. Now Paula has 34 grapes. How many grapes did Paula have to start with?

I think the answer is 52




Stefan

I think the answer is 16



Tasha

I don't think the answer is 52 or 16



Emma

Circle the name of the student you agree with. Use words or pictures to show your thinking.

UNCOVERING STUDENT UNDERSTANDING: USE A PROBE TO UNCOVER UNDERSTANDINGS AND AREAS OF DIFFICULTIES

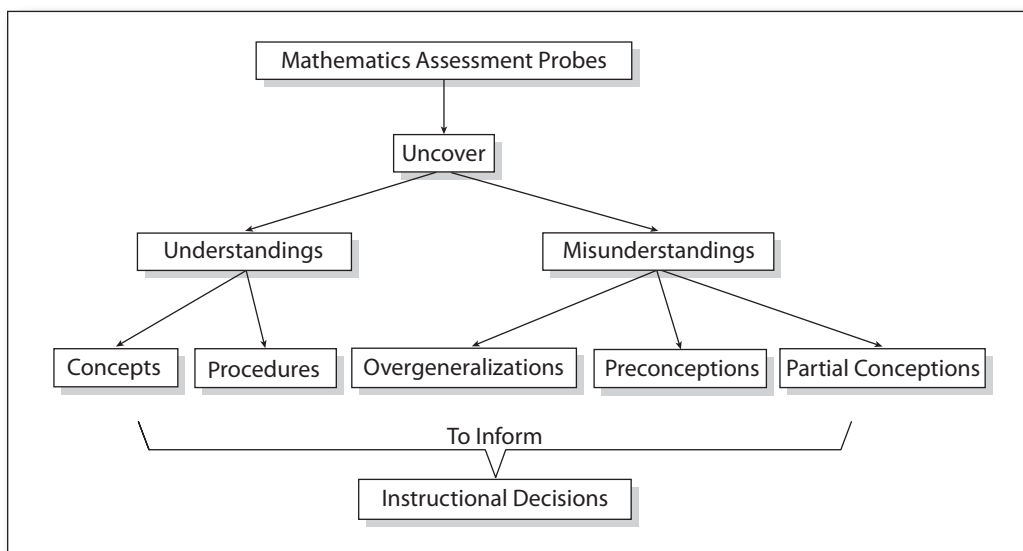
Misunderstandings are likely to develop as a normal part of learning mathematics. These misunderstandings can be classified as conceptual misunderstandings, overgeneralizations, preconceptions, and partial conceptions. These are summarized in Figure 1.6, and each is described in more detail below.

In *Hispanic and Anglo Students' Misconceptions in Mathematics*, Jose Mestre (1989) summarized cognitive research as follows: Students do not come to the classroom as “blank slates” (Resnick, 1983). Instead, they come with theories constructed from their everyday experiences. They have actively constructed these theories, an activity crucial to all successful learning. Some of the theories that students use to make sense of the world are, however, incomplete half-truths (Mestre, 1987). They are misconceptions.

Misconceptions are a problem for two reasons. First, when students use them to interpret new experiences, misconceptions interfere with learning. Second, because they have actively constructed them, students are emotionally and intellectually attached to their misconceptions. Even when students recognize that their misconceptions can harm their learning, they are reluctant to let them go. Given this, it is critical that primary teachers uncover and address their students' misconceptions as early as possible.

For the purposes of this book, misconceptions will be categorized as *overgeneralizations*, *preconceptions*, *partial conceptions*, and *conceptual*

Figure 1.6 Mathematics Assessment Probes



Source: Adapted from Rose, Minton, & Arline (2007).

misunderstandings. The following brief summary describes each of these categories of misconception.

- **Preconceptions:** Ideas students have developed from previous experiences, including everyday interactions and school experiences. Often preconceptions are accurate at the level of mathematics experience but could be an issue if students do not consciously integrate new mathematical ideas.
- **Overgeneralizations:** Information extended or applied to another context in an inappropriate way. This also includes vernacular issues related to differences between the everyday meanings of words and their mathematical meanings.
- **Partial Conceptions:** Hybrids of correct and incorrect ideas. This may result from difficulty generalizing or connecting concepts or distinguishing between two concepts.
- **Conceptual Misunderstandings:** Content students “learned” in school but have misinterpreted and internalized and that often goes unnoticed by the teacher. Students often make their own meaning out of what is taught. (Above categories adapted from Keeley, 2012)

Table 1.2 provides an example from each of the above categories. The examples provided are from progressions for the Common Core State Standards in Mathematics written by the Common Core Standards Writing Team (2011a, 2011b).

Some misunderstandings do not fall distinctly into one category but can be characterized in more than one way. For example, the conceptual misunderstanding of the equal sign as “the answer is” can also be considered an overgeneralization. In addition, some misconceptions are more deeply rooted and difficult to change than others. It is important to make the distinction between what we might call a silly mistake and a more fundamental one, which may be the product of a deep-rooted misunderstanding. In her guest editorial titled “Misunderstanding Misconceptions,” Page Keeley described various practitioner misunderstandings related to using the Science Probes in the National Science Teachers Association’s *Uncovering Student Ideas in Science* series (Keeley, 2012). Both in our work with Page and with mathematics educators using the *Uncovering Student Thinking in Mathematics* resources, we have encountered many similar misunderstandings among teachers:

- *All misconceptions are the same.* The word *misconception* is frequently used to describe all ideas students bring to their learning that are not completely accurate. In contrast, researchers often use labels such as *alternative frameworks*, *naïve ideas*, *phenomenological primitives*, *children’s ideas*, et cetera, to imply that these ideas are not completely “wrong” in a student’s common-sense world.
- *Misconceptions are a bad thing.* The word *misconception* seems to have a pejorative connotation to most practitioners. According to constructivist theory, when new ideas are encountered, they are either accepted, rejected, or modified to fit existing conceptions. It is the

Table 1.2 Misconceptions: Categories and Examples

<i>Misconception Category</i>	<i>Example</i>
Preconceptions: Ideas students have from previous experiences, including everyday interactions	<ul style="list-style-type: none"> • Students usually know or can learn to say the counting words up to a given number before they can use these numbers to count objects or to tell the number of objects. Students become fluent in saying the count sequence so that they have enough attention to focus on the pairings involved in counting objects (p. 4).
Overgeneralizations: Extending information to another context in an inappropriate way	<ul style="list-style-type: none"> • When counting two sets of objects, students learn that even if one group looks as if it has more objects (e.g., has some extra sticking out), matching or counting may reveal a different result (p. 5). • The language of comparisons can be difficult. For example, “Julie has three more apples than Lucy” tells both that Julie has more apples and that the difference is three. Many students “hear” the part of the sentence about who has more, but do not initially hear the part about how many more. Another language issue is that the comparing sentence might be stated in either of two related ways, using “more” or “less” (p. 12).
Partial Conceptions: Using some correct and some incorrect ideas. This may result from difficulty generalizing or connecting concepts or distinguishing between two concepts.	<ul style="list-style-type: none"> • Students understand that the last number name said in counting tells the number of objects counted. Prior to reaching this understanding, a student who is asked “How many kittens?” may regard the counting performance itself as the answer, instead of answering with the cardinality of the set (p. 4). • The make-a-ten methods are more difficult in English than in East Asian languages because of the irregularities and reversals in the teen number words (p. 16).
Conceptual Misunderstandings: Content that students “learn” in school but have misinterpreted and internalized and that often goes unnoticed by the teacher. Students often make their own meaning out of what is taught.	<ul style="list-style-type: none"> • Equations with one number on the left and an operation on the right (e.g., $5 = 2 + 3$ to record a group of 5 things decomposed as a group of 2 things and a group of 3 things) allow students to understand equations as showing in various ways that the quantities on both sides have the same value (p. 10). Students who only see equations written in one way often misunderstand the meaning of the equal sign and think that the “answer” always needs to be to the right of the equal sign.

cognitive dissonance students experience when they realize an existing mental model no longer works for them that makes students willing to give up a preexisting idea in favor of a scientific one. Having ideas to work from, even if they are not completely accurate, leads to deeper understanding when students engage in a conceptual-change process (Watson & Konicek, 1990).

- *All misconceptions are major barriers to learning.* Just as some learning standards have more weight in promoting conceptual learning than others, the same is true of misconceptions. For example, a student may have a misconception for only one type of problem situation (see Figure 1.5, Solving Number Stories) but can make great strides in learning to model and represent operations for other situations (adapted from Keeley, 2012).

To teach in a way that avoids creating any misconceptions is not possible, and we have to accept that students will make some incorrect generalizations that will remain hidden unless the teacher makes specific efforts to uncover them (Askew & Wiliam, 1995). Our job as educators is to minimize the chances of students' harboring misconceptions by knowing the potential difficulties students are likely to encounter, using assessments to elicit misconceptions and implementing instruction designed to build new and accurate mathematical ideas.

The primary purpose of the Probes is to elicit understandings and areas of difficulties related to specific mathematics ideas. In addition to these content-specific targets, the Probes also elicit skills and processes related to the Standards for Mathematical Practices, especially those related to use of reasoning and explanation. If you are unfamiliar with the Standards for Mathematical Practices, descriptions of them can be found in Appendix A.

WHAT IS THE STRUCTURE OF A PROBE?

Each Probe is designed to include two levels of response, one for elicitation of common understandings and misunderstandings and the other for the elaboration of individual student thinking. Each of the levels is described in more detail below.

Level 1: Answer Response

Since the elicitation level is designed to uncover common understandings and misunderstandings, a structured format using stems, correct answers, and distractors is used to narrow ideas found in the related research. The formats typically fall into one of four categories, shown in Figures 1.7 through 1.10.

Selected Response

- Two or more items are provided, each with one stem, one correct answer, and one or more distractors.

Math Talk Probe

- Two or more statements are provided, and students choose the statement they agree with. This format is adapted from *Concept Cartoons in Science Education*, created by Stuart Naylor and Brenda Keogh (2000) for probing student ideas in science.



Figure 1.7 Chicken and Eggs Multiple Selections Probe Problems 3 and 4

Nina's Eggs

Nellie's Eggs

Who has more eggs? Circle the letter.

A. Nina has more eggs. B. Nellie has more eggs.
 C. Nina and Nellie have the same number of eggs.

Tati's Eggs

Meena's Eggs

Who has more eggs? Circle the letter.

A. Tati has more eggs. B. Meena has more eggs.
 C. Tati and Meena have the same number of eggs.

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Examples and Nonexamples Card Sort

- Several examples and nonexamples are given, and students are asked to sort the items into the correct piles.

Justified List

- Two or more separate problems or statements are provided, and students must justify each answer they choose as correct.

Level 2: Explanation of Response Choice

The second level of each of the Probes is designed so students can elaborate on the reasoning they used to respond to the Level 1 elicitation question. Mathematics teachers gain a wealth of information by delving into the



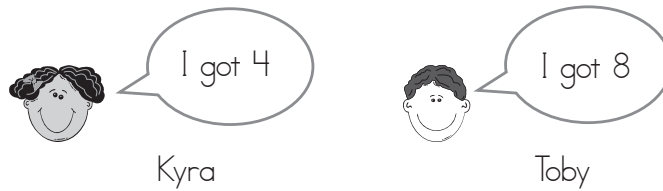
Figure 1.8 Comparing Measures Math Talk Probe

Comparing Measures

Two students were asked to measure the length of a book using either an eraser or a paperclip. The picture shows how these items compare in size.



Kyra and Toby both measured the same book using one of the items from the picture above.



If both of them are correct, what items did they measure with?

	Circle One	
Kyra:	eraser	paperclip
	Circle One	
Toby:	eraser	paperclip
Explain your choices.		

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thinking behind students’ answers, not just when answers are wrong but also when they are correct (Burns, 2005). Although the Level 1 answers and distractors are designed to target common understandings and misunderstandings, the elaboration level allows educators to look more deeply at student thinking. Often a student chooses a specific response, correct or incorrect, for a typical reason. Also, there are many different ways to approach a problem correctly; therefore, the elaboration level allows educators to look for trends in thinking and in methods used. At the early grades, much of this elaboration is done through verbal exchanges with students while administering the Probe, shifting to written elaborations as students develop the ability to write them. Chapter 7 delves deeper into expectations for this elaboration and its relationship to the Common Core Mathematical Practices.



Figure 1.9 Is It a Triangle? Examples and Nonexamples Card Sort

Triangle	NOT a Triangle		

Advance Preparation: Create cards by photocopying on card stock and cutting. Separate the two blank cards and the two label cards from the deck, and shuffle the rest of the cards.

Instructions:

1. Invite the student(s) to sort the cards into two piles: Triangle and NOT a Triangle. Use the label cards to identify the piles.
2. As students finish the sort, give them the blank cards, and ask them to create their own Triangle and NOT a Triangle cards.
3. Ask students to choose three cards from the Triangle pile (or choose three cards for them). Ask them to explain or show how they knew these cards should go in the Triangle pile.
4. Ask students to choose three cards from the NOT a Triangle pile (or choose three cards for them). Ask them to explain or show how they knew these cards should go in the NOT a Triangle pile. Use the recording sheet as appropriate.

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QUEST CYCLE: STRUCTURE OF THE SUPPORTING TEACHER NOTES

The Teacher Notes, included with each Probe, have been designed to help you prepare for a QUEST Cycle. The first two components of the cycle, *determining questions around the key mathematics* and *uncovering student understandings and areas of difficulties*, have been described more fully above. We will use the description of the Teacher Notes to provide more details about the remaining components of the cycle.



Questions to Consider About the Key Mathematical Concepts

This section of the Teacher Notes helps to focus a teacher on the key conceptual and procedural mathematics addressed by the particular Probe and gives information about alignment to Common Core standards at a particular grade level. Figure 1.11 shows an example from this section of the Chicken and Eggs Probe Teacher Notes.



Figure 1.10 Are They Equivalent?

1. Without adding the two numbers, use what you know about adding three-digit numbers to decide which of the number expressions below are equivalent to

$$427 + 569$$

	Circle One	Explain Your Answer
A. $724 + 965$	Yes No	
B. $467 + 529$	Yes No	
C. $527 + 469$	Yes No	
D. $472 + 596$	Yes No	
E. $927 + 69$	Yes No	

2. Without subtracting, use what you know about subtracting three-digit numbers to decide which of the number expressions below are equivalent to

$$618 - 498$$

	Circle One	Explain Your Answer
A. $620 - 500$	Yes No	
B. $681 - 489$	Yes No	
C. $608 - 488$	Yes No	
D. $698 - 418$	Yes No	
E. $618 - 418 - 80$	Yes No	

Figure 1.11 Questions to Consider About the Key Mathematical Concepts

Do students apply counting and cardinality when comparing two sets of objects? To what extent do they

- apply rote counting along with an understanding of one-to-one correspondence when they match one object in the set to one count?
- continue through a sequence of counting numbers, “1, 2, 3, 4,” and so on until they’ve counted the whole set?
- answer the question “how many are there?” with the last number they have counted?

Common Core Connection (K.CC)

Grade: Kindergarten

Domain: Counting and Cardinality (CC)

Clusters:

B. Count to tell the number of objects.

K.CC.B.4. Understand the relationship between numbers and quantities; connect counting to cardinality.

K.CC.B.5. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

C. Compare numbers.

K.CC.C.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.



Uncovering Student Understanding About the Key Concepts

This section of the Teacher Notes (Figure 1.12) breaks down the concepts and ideas described in the “Questioning” section into specific understandings and areas of difficulty targeted by the Probe.



Exploring Excerpts From Educational Resources and Related Research

This section of the Teacher Notes (Figure 1.13) includes excerpts from cognitive research related to the common areas of difficulty targeted by the Probe. The excerpts are meant to provide some background from the research base behind the development of the Probe. The references provide an opportunity for you to seek additional information when needed.

This research base is an important component in the Probe development process. More information on the origin of the Probe development process can be found in Appendix B.

Figure 1.12 Uncovering Student Understanding About the Key Concepts

Using the Chicken and Eggs Probe can provide the following information about how the students are thinking about counting and cardinality.

<p><i>Do they</i></p> <ul style="list-style-type: none"> • recognize that the arrangement of a group of objects does not change the count? • give the last number they've counted as the count of the set of objects? • apply one-to-one correspondence, applying just one counting number to each object they count? • understand how to use the count of two sets to compare them, using words such as <i>more</i> or <i>less</i>? 	<p>OR</p> <p>OR</p> <p>OR</p> <p>OR</p>	<p><i>Do they</i></p> <ul style="list-style-type: none"> • think that the arrangement of the eggs determines which is greater? • begin to count all over again when asked how many there are? • skip over eggs when they are counting, or double-count eggs? • compare the size of the eggs or how they are spaced or arranged rather than the quantities?
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Figure 1.13 Exploring Excerpts From Educational Resources and Related Research

Common areas of difficulty for students:

“Counting one past the actual number of items—Young children often have difficulty tagging items (touching and saying a number) and partitioning (moving aside counted items) simultaneously. This often leads to saying one extra number name.” (Bay Area Mathematics Task Force, 1999, p. 10)

Misusing the acoustical sequence of numbers. Instead of counting per word (numeral), they count per syllable. For example, sev-en means 2 objects; e-lev-en means 3 objects. (Van Den Brink, 1984, p. 2)

Thinking sets of objects that are spread out have a larger count than those that are arranged close to one another. Students are “misled by perceptual clues—six items spread out may appear to be more than 7 items close together.” (Bay Area Mathematics Task Force, 1999, p. 10)

When a student can count 4 objects (1, 2, 3, 4) and can answer “4” when asked “How many are there?” the student has developed cardinality. Children who understand the short cut to describing the count of a set by using the last number of the enumeration of the count (4) rather than repeating the whole count (1, 2, 3, 4) are said to have grasped the *cardinality principle*. (Gelman & Gallistel, 1978; Schaeffer, Eggleston, & Scott, 1974)



Surveying the Prompts and Selected Responses in the Probe

This section of the Teacher Notes (Figure 1.14) includes information about the prompt, selected response/answer(s), and the distractors. Sample student responses are given for a selected number of elicited understandings and misunderstandings. This initial preparation will help expedite the analysis process once you administer the Probe to students.

Figure 1.14 Surveying the Prompts and Selected Responses in the Probe

There are four cards, each containing two sets of objects to compare. The items are designed to elicit understandings and common difficulties as described below.

Carla and Bonnie

<i>If a student chooses</i>	<i>It is likely that the student</i>
Carla has more eggs	<ul style="list-style-type: none"> thinks that the arrangement of a set of objects is related to the size or count of the set of objects (more spread out or random is larger).
Bonnie has more eggs	<ul style="list-style-type: none"> counts incorrectly or has difficulty with comparison words like <i>same</i> or <i>more</i>.
Carla and Bonnie have the same number of eggs (correct answer)	<ul style="list-style-type: none"> applies one-to-one correspondence and other counting strategies and is able to compare quantities using <i>same</i> and <i>more</i>.

Penny and Dee Dee

<i>If a student chooses</i>	<i>It is likely that the student</i>
Penny has more eggs	<ul style="list-style-type: none"> is applying accurate counting and comparison strategies.
Dee Dee has more eggs	<ul style="list-style-type: none"> has a misconception that objects that are spread out in an arrangement are “more” than objects arranged more closely to one another. (See Sample Student Response 1, Figure 1.16.)
Penny and Dee Dee have the same number of eggs	<ul style="list-style-type: none"> has made a counting error, such as missing or double-counting an egg.



Teaching Implications and Considerations

Being aware of student difficulties and their sources is important, but acting on that information to design and provide instruction that will

diminish those difficulties is even more important. The information in this section of the Teacher Notes (Figure 1.15) is broken into two categories: (1) ideas for eliciting more information from students about their understanding and difficulties, and (2) ideas for planning instruction in response to what you learned from the results of administering the Probe. Although these ideas are included in the Teacher Notes, we strongly encourage you to pursue additional research-based teaching implications.

Figure 1.15 Teaching Implications and Considerations

Ideas for eliciting more information from students about their understanding and difficulties:

- How can you tell that _____ has more eggs than _____?
- Does it matter which egg you start with when you count?
- Is there more than one way to determine who has more eggs?
- What happens when some eggs are bigger than other eggs? (Refer to Nina and Nellie or Tati and Mina card.)
- How do you count the eggs when they are in a line? In rows? In a mixed up jumble? (Refer to Carla and Bonnie and Penny and Dee Dee cards.)

Ideas for planning instruction in response to what you learned from the results of administering the probe:

- Use concrete materials. Skill in counting is supported by providing sets of blocks or counters that students can manipulate as they are counting. These concrete materials can help to build understanding of one-to-one correspondence and provide engaging practice in matching number names with the objects being counted. Counting objects arranged in one straight row is easier for children than counting objects arranged randomly or in organized rectangular array or circles.
- Provide opportunities for students to build or draw sets of different sizes to build understanding of comparative terminology. Ask students to create two groups of counters, one that is more than the other. Give students practice building sets that fit certain criteria for comparison: the same, more, or less.
- Be explicit about counting guidelines—each object must be counted once and only once—and discuss strategies for counting. How do you keep track of items you have already counted? Does it matter where you start when you are counting a set? How do you decide where to start? Do you use any shortcuts when you are counting a set of objects?
- Write numbers to show the counts of sets of objects: Students need experiences in connecting the number name with its numeral representation. Connecting the last number counted with its numeric representation can support the idea of cardinality—I count up until I've counted each object, and the last name I count is the number of objects. I can describe the count by saying or writing a number.

Included in the Teaching section of the Teacher Notes are sample student responses; examples of these are shown in Figure 1.16.

Figure 1.16 Sample Student Responses to Chicken and Eggs Probe**Responses That Suggest Difficulty***Sample Student Response 1*

Student: Dee Dee has more eggs than Penny.

Teacher: How do you know?

Student: Dee Dee's eggs go all the way to here (points to the last egg on the right) and Penny's go to here (points to the last egg on the right).

Teacher: How many eggs does Dee Dee have?

Student: 7

Teacher: And how many eggs does Penny have?

Student: 8

Teacher: And Dee Dee has more eggs.

Student: Yes.

Sample Student Response 2

Student: Nina has more eggs.

Teacher: Nina has more eggs than Nellie?

Student: Yes.

Teacher: How do you know?

Student: The eggs are bigger?

Teacher: Yes, the eggs are bigger. Are there more eggs here (pointing to Nina's eggs) than here (pointing to Nellie's eggs)?

Student: Yes.

Responses That Suggest Understanding*Sample Student Response 3*

Student: (Pointing to Tati's eggs and counting) 1, 2, 3, 4, 5, 6, 7. 7 eggs.
(Pointing to Meena's eggs and counting) 1, 2, 3, 4, 5, 6, 7, 8. 8 eggs.


Teacher: Does Tati have more eggs? Does Meena have more eggs? Or do they have the same number of eggs?

Student: Meena has more eggs.

Teacher: Why do you say Meena has more eggs?


Student: She has 8 eggs, and 8 is 1 more than 7.

Figure 1.17 Reflection Template




Questions to Consider About the Key Mathematical Concepts

What is the concept you wish to target? Is the concept at grade level, or is it a prerequisite?




Uncovering Student Understanding About the Key Concepts

How will you collect information from students (e.g., paper and pencil, interview, student response system, etc.)? What form will you use (e.g., one-page Probe, card sort, etc.)? Are there adaptations you plan to make? Review the summary of typical student responses.




Exploring Excerpts From Educational Resources and Related Research

Review the quotes from research about common difficulties related to the Probe. What do you predict to be common understandings and/or misunderstandings for your students?



Surveying the Prompts and Selected Responses in the Probe

Sort by selected responses; then re-sort by patterns in thinking. What common understandings/misunderstandings did the Probe elicit? How do these elicited understanding/misunderstandings compare to those listed in the Teacher Notes?



Teaching Implications and Considerations

Review the bulleted list, and decide how you will take action. What actions did you take? How did you assess the impact of those actions? What are your next steps?

Variations

For some Probes, adaptations and variations are provided and can be found following the Teacher Notes and sample student responses to the Probe. A variation of a Probe provides an alternate structure (selected response, multiple selections, opposing views, or examples/nonexamples) for the question within the same grade span. In contrast, an adaptation to a Probe is similar in content to the original, but the level of mathematics is changed for a different grade span.

Action Research Reflection Template

A reflection template is included in Appendix C. The reflection template provides a structured approach to working through the QUEST cycle with a Probe. The components of the template are described in Figure 1.17.

BEGINNING TO USE THE PROBES

Now that you have a background on the design of the Probes, the accompanying Teacher Notes, and the QUEST Cycle, it is time to think about how to get started using the Probes with your students.

Choosing a Probe: Determining which Probe to use depends on a number of factors, including time of year, alignment to curriculum, and range of abilities within your classroom. We recommend you spend some time reviewing the Probes at your grade level first but also make note of additional Probes that may be appropriate for your students.

Deciding how to administer a Probe: Depending on your purpose, Probes can be given to one student or to all students in your classroom. You may wish to give a Probe to only one student (or several) if you notice the student or group is struggling with a related concept. By giving a Probe to all students, you can gain a sense of patterns of understanding and difficulty in order to target instruction. All Probes can be given as verbal interviews, and many of the kindergarten Probes are written as verbal interviews, but we encourage you whenever appropriate to ask students to write and/or draw their responses instead of explaining them verbally. Many teachers script above what the students have written, a practice that students may already be familiar with from their writing instruction. Scripting is a handwritten record of the student's spoken explanation and the teacher's related notes. Chapter 7 includes additional instructional considerations.

Talking with students about Probes: We have found that young students are very much able to understand the diagnostic nature of the Probes, especially if the process is shared explicitly with them. Talk to your students about the importance of explaining their thinking in mathematics and why you will ask additional questions to understand more about their thinking.

When giving a Probe, be sure to read through the directions, repeating them as necessary. Do not try to correct students on the spot; instead, ask

additional probing questions to determine whether the additional questions prompt the student to think differently. If not, do not stop to try to teach the students “in the moment.” Instead, take in the information and think about the next appropriate instructional steps. If students are having difficulty, reassure them that you will be working with them to learn more about the content in the Probe.

HOW TO NAVIGATE THE BOOK

This chapter provided the background information needed to begin to dig into the Probes and think about how you will use them with your students. The next five chapters include 20 sets of Probes and accompanying Teacher Notes, and the final chapter includes additional considerations for using the Probes.

Chapters 2 Through 6: The Probes

Many of the mathematics assessment Probes included in this book fall under the topic of number and operations, because the cognitive research is abundant in these areas (Clements & Sarama, 2004), and the Common Core places a strong emphasis on number and operation concepts at grades K–2. Figure 1.18 provides an “at a glance” look at the targeted grade span and related domain of the content of the Probes.

The beginning of each Probe chapter (Chapters 2–6) includes background on the development of the Probes to align with the relevant Common Core domain and standards and a summary chart to guide your review and selection of Probes and variations to use with your students.

Chapters 7: Additional Considerations

The QUEST Cycle components are explained in detail within this chapter as well as for each specific Probe through the accompanying Teacher Notes. In addition to these “specific to the Probe” ideas are instructional considerations that cut across the Probes. Such considerations include ways to use the Probes over time to promote mathematical discussions, support and assess students’ ability to provide justification, and promote conceptual change.

We recommend that you scan the contents of Chapter 7 before beginning to use the Probes but that you not to try to “do it all” the first time out. After experiencing the use of the Probes, return to Chapter 7 to pinpoint one or two considerations to implement.

FINAL CHAPTER 1 THOUGHTS

We hope these Probes will support you in your work in trying to uncover your students’ thinking and understanding and will inspire you to explore ways to respond to their strengths and difficulties in order to support students in moving their learning forward.

Figure 1.18 Mathematics Assessment Probes

<i>Kindergarten</i>			
<i>Chapter</i>	<i>Page Numbers</i>	<i>Probe</i>	<i>CCSS Domain</i>
2	p. 29	Chicken and Eggs	Counting and Cardinality
2	p. 38	Name the Missing Number	Counting and Cardinality
2	p. 46	Dots and Numerals: Card Match	Counting and Cardinality
2	p. 54	Counting and Combining	Counting and Cardinality
2	p. 61	Comparing Numbers	Number and Operations in Base Ten
6	p. 150	Is It a Triangle?	Geometry
6	p. 159	Is It Two Dimensional or Three Dimensional?	Geometry
<i>Grade 1</i>			
3	p. 69	What's the Value of the Digit?	Number and Operations in Base Ten
4	p. 103	Apples and Oranges	Operations and Algebraic Thinking
4	p. 109	Sums of Ten	Operations and Algebraic Thinking
4	p. 118	Completing Number Sentences	Operations and Algebraic Thinking
5	p. 130	Length of Rope	Measurement and Data
6	p. 165	Odd Shape Out	Geometry
6	p. 170	Coloring One Half	Geometry
<i>Grade 2</i>			
3	p. 79	Building Numbers	Number and Operations in Base Ten
3	p. 88	Labeling the Number Line	Number and Operations in Base Ten
3	p. 93	Are They Equivalent?	Geometry
4	p. 123	Solving Number Stories	Operations and Algebraic Thinking
5	p. 137	Comparing Measures	Measurement and Data
5	p. 143	Reading Line Plots	Measurement and Data