CHAPTER 1

Introduction

Some key concepts

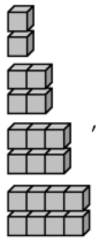
- This book is about teaching mathematics to learners who have learning *differences*, not learning *difficulties*.
- Children who think and learn visually and actively often struggle with a school curriculum that relies heavily on print.
- Visual and kinaesthetic mathematics are under-valued in the classroom.
- The development of 'pictures in the mind' can help all learners to understand key mathematical concepts.
- A visual and kinaesthetic approach is worthwhile only if it is based on understanding.

a) Differences and Difficulties

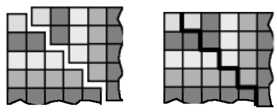
This book is about teaching mathematics to learners who take in information and ideas visually and actively. These learners have learning *differences*, not learning *difficulties*. Teaching and learning in our schools has always been, and continues to be, heavily reliant on print. Literacy is all, whether learners are working from a book or with a screen: other ways of thinking – visual, kinaesthetic, practical – are discounted in the classroom. To become teachers, students must jump over a long series of hurdles, formal and informal, at school, at college and at university. These hurdles consist of print-based activities and assessments that demand a high level of linguistic and symbolic thought but take little account of other ways of thinking and learning. As a result, teachers are rarely selected for their visual or kinaesthetic abilities as these have little impact on their academic achievement. It is their verbal and numerical skills that have opened the doors to success, not their spatial skills. This may make it difficult for teachers to recognise spatial ability in their learners, so real strengths and aptitudes are neglected as children are forced to struggle with a curriculum which is largely presented through printed materials that they find hard to access.

Because the curriculum is so heavily print-based, 'proper' school mathematics is defined by what can be printed in a book, and preferably in text. Definitions and proofs that depend on models or dynamic geometry rather than on symbols are second best. So, for example,

The number seven is not  or , it is the symbol 7

The 2 times table is not , it is the symbols

$2 \times 1 = 2$
$2 \times 2 = 4$
$2 \times 3 = 6$
$2 \times 4 = 8$

The formula for 1 + 2 + 3 + … n is not , it is the symbols $\frac{n^2 + n}{2}$

Pictures and models may be used to support learning, especially in the early stages, but the end point is symbolic. Symbols are easier to print, and they always take precedence over visual or kinaesthetic representations.

But to some children, the numbers and symbols on the page are just squiggles. These learners can *see* that seven is five plus two, or that twice two two's will fit together to make four two's, or that the sum of the first n counting numbers is half the area of a rectangle with sides n and $n + 1$. They may not be able to put it into words, but they can see it, and perhaps draw it. But the printed symbols, the squiggles, are meaningless. It is for these children, and for their teachers, that this book and CD have been put together.

b) Visual, Auditory, Kinaesthetic

So – what is 'visual and active mathematics'? How can we go about teaching 'visually and actively'?

There are nearly as many theories about learning as there are researchers writing about it. Steve Chinn offers a useful summary of 'thinking styles in mathematics', and shows how, to some extent at least, the different models overlap and interrelate (Chinn, 2004: 59–75). But for general classroom use the VAK model – Visual, Auditory, Kinaesthetic – will serve us well. It is at least as old as Confucius –

I hear, and I forget;

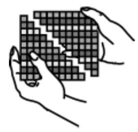
I see, and I remember;

I do, and I understand.

This model is quite straightforward – and it works in real classrooms, not just in lecture halls – so it can provide the theoretical structure we need for the ideas and activities discussed in this book.

The phrase *kinaesthetic learning* is sometimes taken to mean any activity that involves the use of apparatus. This may be considered particularly appropriate for 'slow learners' – at least they will have something to *do* in their mathematics lessons. But if the focus of the teaching is primarily on the correct use of the apparatus, rather than on the mathematical understanding that the apparatus is designed to develop, then it may have very limited impact. Learners will just follow the instructions to use the equipment, without necessarily relating what they are doing to mathematics.

Kinaesthetic learning calls for a lot more than a pile of cubes or a pair of scissors and some card. It involves using your whole being, engaging all your senses to feel or imagine what is happening. Visual, aural and kinaesthetic learning are all intertwined: together they can lay down a memory – of movement, feeling, sight and sound – that will be recalled as a total experience, not just as a recited chant. For example, when I think about the number seven I can *feel* the seven in all the fingers of one hand and two fingers of the other. When I factorise, I can imagine pulling apart *eight* to make two sets of *two twos*. And I can feel myself breaking up a 10 by 11 unit rectangle into two halves to find the sum of the first ten counting numbers, 1 + 2 + 3 + … + 10. Because I have done all these activities, and have understood the mathematics that they represent, I do not need to actually hold up my fingers or make blocks of cubes in order to recall them. But what I recall is most certainly not a chant or a formula: it is more like a moving picture – a sort of waking dream. This, I believe, is kinaesthetic learning.



It is essential, too, that visual images are fully supported by kinaesthetic movement. Computers have provided an incredibly powerful tool that enables us to present our ideas visually, with dynamic images that can convey key concepts meaningfully and memorably. But there may be a temptation to do away with the physical equipment altogether – the piles of cubes, the scissors and card, and all the other bits and pieces that help us to create and share representations of key mathematical concepts. After all, we have an unlimited supply of cubes on the screen, and they never roll off the table and get lost. It would be much more straightforward to rely on those. But our learners need the real cubes. They need to handle them, and to actually create and break up the models for themselves. You will find lots of dynamic images in the files on the CD that accompanies this book, and they should be helpful. But they will not – they cannot – replace the contents of the equipment cupboard. You need both.

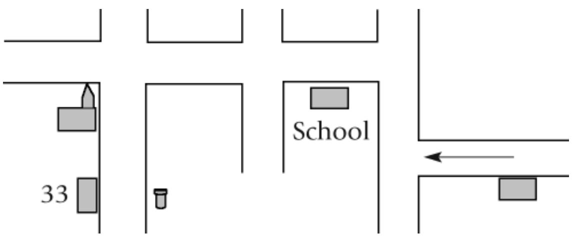
All the learners in a mathematics classroom – like all the teachers – are able to think visually and kinaesthetically to a greater or lesser degree. There is not a clear-cut divide between spatial thinkers and those who think in words and symbols. The chief difference lies, not in the ability of different learners to think spatially or numerically, but in the value that is placed on the different ways of thinking. But how can teachers spot visual and kinaesthetic ability, and identify learners who are likely to learn more effectively through models that they can construct and take apart, and through 'pictures in the mind'? Teachers may well notice the visual and kinaesthetic thinkers in their classroom by their responses to different types of mathematical task. These are the learners who have found all the nets of a cube before most of the rest of the class have grasped what a net is – but for whom 'seven eights' are 'forty-three' on Tuesday, and 'sixty-two' on Wednesday. With a print-based curriculum they rarely shine – but just occasionally they take everyone (including, quite possibly, themselves) by surprise with their ability to just *see* the solution to a problem with which other learners are struggling.

Any teaching idea, no matter how inspirational, can be reduced to 'rote learning' – *I hear and I forget*. On the other hand, the dreariest exercise might be transformed into a basis for real understanding by a teacher who can unpack the underlying concepts and help learners to understand and use them. The most effective mathematical thinkers are flexible: they try different approaches to the problem in hand, finding out what works best and relating each new idea to what they already know. The *hearing*, the *seeing* and the *doing* support one another, as the pictures, models and activities give meaning to the spoken or written definitions and procedures. Learners may adopt different ways of thinking as they first explore and understand, and then rehearse and apply, each new concept. But for learners who think more easily in pictures and movement than in words and symbols, *seeing* and *doing* may offer access to key mathematical ideas, while too much time spent *hearing* may slam the door shut.

c) Pictures in the Mind

Some people can follow a set of directions easily, but others find it much more helpful to have a visual image. For example, one person might find it easy to follow a written description of a route:

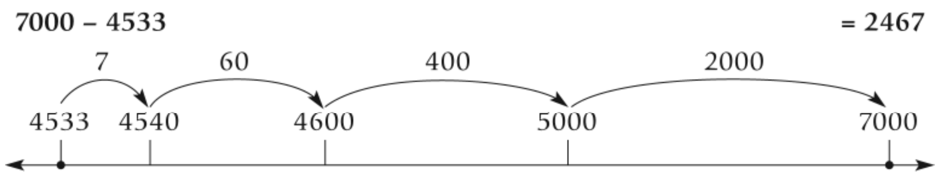
> *Turn left out of the gate, and walk to the T-junction at the end of the road. There you should cross the road and turn right. Take the first left turn, and walk past the school and across the crossroads. You will come to another crossroads, with a church on the corner; there you must turn left. Walk about fifty metres down the road, and the house you want, number 33, is on the right opposite the post box.*



But another might prefer a map. They find the map easier than the linear series of instructions to understand and to follow, and they can recall it more easily when they need to find their way again along the same route.

In the mathematics classroom diagrams may be used, but, as we have seen, they are generally subservient to the written, symbolic forms. A map (or its equivalent) is rarely considered to be enough on its own – while a written formula, or a set of rules for carrying out a procedure, can stand alone. Learners who can take in and remember a series of instructions, or a formula, or the 'rules' for adding fractions or finding the sine of an angle, achieve high grades and feel successful. But those learners for whom such rules and procedures seem meaningless have great difficulty recalling them, and cannot use them efficiently to solve problems. They may struggle to make sense of the symbols and instructions – or they may just give up in despair. Either way, they do not achieve any real understanding of the concepts that underlie the routines and methods that they are trained to use.

The main purpose of any model or image is to develop the learners' understanding, so they do not just learn *how* to use a method to solve a problem, but they also understand *why* it works. For example, the image of a number line may help some learners to see a subtraction as finding the 'distance' between two numbers.



This approach may make much better sense than a standard algorithm –

Nought take away three, I can't, borrow one, I can't, borrow one, I can't, borrow one, cross out the seven and put six, make ten in the next column, cross out the ten and make nine, make ten in the next column, cross out the ten and make nine… and so on.

	6 9 9
7 0 0 0	~~7~~ ¹~~0~~ ¹~~0~~ ¹0
4 5 3 3	4 5 3 3
	2 4 6 7

The number line offers far more than this sequential set of 'rules' for getting the right answer. The picture itself – whether printed, drawn, seen on screen, or just imagined – carries within it an explanation of why the method works. In this way, mathematical ideas from the simplest to the most complex can be made manifest, and so become meaningful and memorable to all our learners – not just to those who struggle to make sense of the printed text.

But the number line, like any other 'model to think with', could be used as just another routine, to be learnt by rote and followed blindly without any understanding of the meaning of each step. Used like this it will be no more helpful, and it will be considerably less tidy, than a numerical algorithm. This book, and the accompanying CD, offer a range of models and images that may be useful, particularly for learners who think more easily in pictures than in words and symbols. By themselves, however, learnt as yet more methods and routines, these models will be useless. If some learners can, and if they really must, learn and recall mathematics without understanding then they will do better to acquire the numerical and symbolic routines. These are generally shorter, neater, and easier to memorise and apply than the pictures and models exemplified in this book. For visual and kinaesthetic thinkers, however, this is not an option. They must understand the mathematics that they are taught. Otherwise they may perhaps learn – but they will almost inevitably forget.

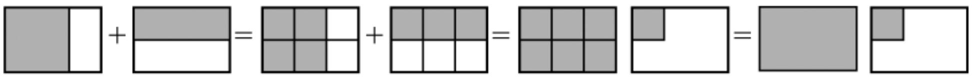
d) Using Symbols and Understanding Diagrams

Our single most important function as maths teachers is to develop our learners' understanding of mathematics. Using mathematical language, manipulating numbers and symbols, applying mathematics to solve problems – all this comes into it, of course. But the basis, the rock on which mathematics education is built, is understanding.

Unfortunately, it is terribly easy to teach learners how to manipulate symbols without understanding. Any teacher with a little determination can teach *how* to add fractions, or *how* to find the area of a circle, or whatever. Learners can learn to get 'right answers' using symbols and the rules for combining them with little understanding of what they mean. Those who can manipulate symbols quickly and efficiently are often thought to be working at a 'higher level' than those who use diagrams or equipment to work through a problem, making sense of each step on the way. A learner who writes

$$\frac{2}{3} + \frac{1}{2} = \frac{2}{3} \times 2 + \frac{1}{2} \times 3 = \frac{4}{6} + \frac{3}{6} = \frac{7}{6} = 1\frac{1}{6}$$

may be rated much more highly that one who uses a more meaningful graphical approach,



But a learner who just goes through the steps, and cannot explain *why* $^2/_3$ is equal to $^4/_6$, and $^4/_6 + {^3/_6}$ is equal to $^7/_6$ which is equal to $1\,^1/_6$, is not working at a higher level than a learner who can use, understand and explain the diagrams. These lead, not just to the 'right answer', but to an explanation – a sort of proof that $^2/_3 + {^1/_2}$ really does equal $1\,^1/_6$. This involves much more mathematics than any rote learning of meaningless symbolic manipulation. Written numbers and symbols are valuable, and indeed essential, tools for mathematics, but we must always ensure that they are used to express, support and communicate mathematical understanding, not to take its place.

e) Activities and Images – Using the CD

The CD which accompanies this book offers a range of resources, including twenty-odd dynamic PowerPoint presentations showing how models and images may be used to help learners to develop their understanding of key mathematical concepts. You will need the 2003 or later version of PowerPoint to see these. It is important that teachers follow the instructions carefully, clicking the mouse only when there is a small circle in the bottom right-hand corner of the screen. Clicking too soon will cause the program to move on to the next screen, often skipping the animations which are key to the concepts being presented. Pausing provides an opportunity for learners to discuss what is going on and to visualise the movement of the models before they watch the animations on the screen. Teachers will need to play through the presentations that they are going to use in advance so that they can identify the points where they can most usefully pause for comments, visualisation and discussion.

But it should always be remembered that there is not just one model that will work for every learner – there are many possibilities. The chapters that follow, and the accompanying CD, offer a range of suggestions which relate to a variety of topics at different levels, but teachers may well have others that work better in their classrooms. The ideas put forward here are intended primarily as illustrations of an approach – an approach that seeks out *models to think with* that can help learners to develop their understanding. Some of these ideas may be useful for particular learners, but they are only a start. Teachers – and the learners themselves – need to be constantly alert, on the lookout for images and models that will represent and explicate specific concepts. You can start with practically any resource or activity, and see how it could be adapted for visual and kinaesthetic learners. It is the approach that matters, not the details of particular activities or materials. Making mathematical concepts manifest with pictures and models will help all learners – even those who could, if it were really demanded of them, learn and remember routines for getting 'right answers'.

Further Reading and Resources

Paul Black and Dylan Wiliam, 1998: *Inside the Black Box*. King's College, London. Available at http://weaeducation.typepad.co.uk/files/blackbox-1.pdf or at http://www.measuredprogress.org/documents/10157/15653/InsideBlackBox.pdf.

This paper has become a classic, inspiring teachers, schools and governments across the world to re-think assessment strategies and practices. It is worth going back to the original to read the clear and accessible exposition of the arguments and evidence on which the changes have been based.

Jo Boaler, 2010: *The Elephant in the Classroom*. Souvenir Press.
In this very readable book Jo Boaler offers a wealth of sound, practical suggestions, based on a secure and well-researched theoretical structure.

Steve Chinn, 2004: *The Trouble with Maths*. Routledge Falmer.
Steve Chinn, 2012: *More Trouble with Maths*. Routledge Falmer.
In these two books Steve Chinn provides straightforward, applicable advice for teachers working with learners who have a range of difficulties with mathematics.