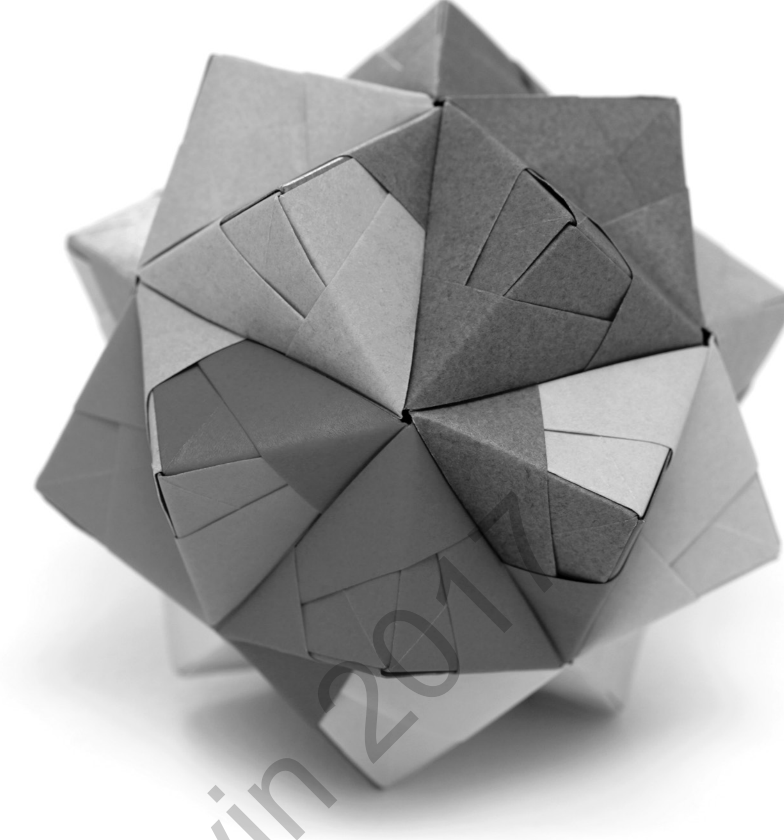

## CHAPTER THREE

# TEACH UP

## MAKING SENSE OF RIGOROUS MATHEMATICAL CONTENT

The second foundational key to effective differentiation is to know the mathematical content in depth. Differentiation will be purposeful and effective only when the mathematics standards are analyzed and form the basis on which instruction and activities are based. In this chapter, you will find:

If I were to ask you how to complete this sentence, what would you say: "The most basic idea in the learning of mathematics is . . . ?" What did you say? Patterns? Number sense? The four mathematical operations? Perhaps you went a different way and said applications. As many times as I ask teachers this question, I receive a wide variety of answers. I have never heard anyone complete the statement in accordance with the original quote, however. According to John Van de Walle (2006), "The most basic idea in the learning of mathematics is . . . mathematics makes sense." Truthfully, I didn't think of that answer the first time I saw this quote either! The more I ponder it, and the more I work with teachers in constructing powerful mathematics lessons, the more I realize that it should be every teacher's mantra. I am just wondering—are there some of you right now thinking that mathematics *doesn't* make sense? Do you have students who would not believe the statement that mathematics makes sense?

This is a chapter on mathematical content, and on trying to make sense of it, which is foundational to differentiation. If we design differentiated tasks before we have clear conceptual understanding, knowledge, and skills of the content we are teaching, we are probably dooming ourselves to a lot of extra planning and sometimes frustrating classroom experiences, with little growth or achievement to show as a result. To paraphrase something Carol Ann Tomlinson once said, "If we are somewhat foggy in what we are teaching, and then differentiate, we end up with differentiated fog." This is not our goal.

## MATHEMATICS MAKES SENSE

Our brains are sense-making machines. In fact, our brains naturally seek patterns and meaning-making to store in our long-term memories (Sousa, 2015). We now know that for an idea or concept to be stored in long-term memory, it needs to make sense and be relevant to the learner. Unfortunately, we often do not teach mathematics as if it is a sense-making subject. We tend to teach skills, or problem types, and then practice, practice, practice . . . until we reach the next skill to be taught. I believe mathematics was the instigator of the phrase "drill and kill." Nevertheless, if we can begin to view mathematics as sense making, we can break this pattern, both for ourselves as teachers and for our students. Most teachers I meet have never been taught to understand mathematics, only how to do it. It makes sense then that many teachers struggle to make mathematics understandable for their students.

We have learned from cognitive science that the human brain is not well designed for memorizing data. It is most efficient and effective when it works with patterns, connections, meaning, and significance—with personal meaning being the most important (Sousa, 2015). Without the necessary time to make sense of learning, students naturally resort to memorization over sense-making. Thus, the most effective way for students to learn mathematics is to prioritize understanding rather than to perform memorization. It is our job to provide lessons that can make it happen.

How do we begin to make sense of mathematics? The first step is to clarify the big ideas that are the foundation for the topic(s) being taught in the unit. Sometimes these essential understandings are embedded in the standard we are addressing, and sometimes they can be seen in some of the exploration tasks in a resource, but they are almost always up to the teacher (or a collaborative team if you are working in one) to determine.

## THEMES AND BIG IDEAS IN MATHEMATICS

Some big ideas in mathematics are true for every grade level, and in fact, they are true for every mathematics course at every level! Figure 3.1 provides a few of these concepts and understandings.

This table is just the beginning of thinking in terms of conceptual understanding in mathematics. Let's take another step. For any mathematical unit based on a group of standards to be taught, the content can be divided into what students will come to know, understand, and be able to do. In the differentiation literature, this is referred to as KUDs or KUDOs (Tomlinson & Imbeau, 2014; Tomlinson & Moon, 2013).

The Know in KUD refers to facts that can be memorized. Our mathematical content is filled with Knows: Mathematical facts, vocabulary, and formulas all fall under the Know category. If you can look it up, it is probably a Know. On the other hand, Understandings

### Consider It!

- What is the difference between knowing, understanding, and doing mathematics?
- What does it look like when students exhibit understanding of mathematics? How is it different from students who know how *to do* mathematics but don't *understand* mathematics?

**FIGURE 3.1**

## GENERAL CONCEPTS AND UNDERSTANDINGS IN MATHEMATICS

| Concept | Understandings |
|---|---|
| Mathematical Operations and Properties | • Each operation in mathematics has meanings that make sense of situations, and the essential meaning of each operation remains true in every context and number system.<br>• Every mathematical operation has specific properties that apply to it, and these properties are the basis for how these operations can and cannot be used.<br>• The properties of operations provide the reasoning for mathematical explanations. |
| Number Sense and Estimation | • Developing mental mathematical strategies that reason about numbers, quantities, and the operations with numbers provides flexibility and confidence in working with numbers.<br>• Estimation allows for establishing the reasonableness of an answer. |
| Units and "Unitizing" | • Determining the "base entity" in a given context or problem (e.g., apples, balloons, miles per hour, or a variable) allows you to make sense of the problem, plan a solution path, and make comparisons.<br>• Units in measurement describe what is being measured, and what is being measured has a specific type of unit. |
| Equality | • An equal sign is a statement that two quantities are equivalent. That equivalency must be maintained throughout any mathematical operations. |
| Shape and Geometry | • Shapes and their properties describe our physical world.<br>• Shapes are categorized and grouped according to their properties.<br>• Relationships among shapes can be described in many ways, including algebraically. |
| Modeling and Representation | • There are many different representations for a given situation, and each representation can provide different aspects of the problem.<br>• Mathematical models represent real-world contexts and provide connections, comparisons, and predictions. |

*Note*: Several concepts presented in this figure came to light during my time working with the high-school teachers of the West Irondequoit School District, Rochester, NY.

are conceptual. They are big ideas and have many layers. Understandings connect the content unit to unit, as well as connect mathematical content to other contents. Understandings also remain true over the years, and it is powerful if the same understandings are used many times to show students clearly how topics are connected. Finally, the Do is what you expect students to be able to Do if they truly Know and Understand. Be careful not to list specific task activities (make a collage or questions 3–8) in the Do category. You are looking for the mathematics within any task that indicates knowledge and understanding. The Do will always start with a verb. To push for understanding, be sure to include high-level verbs from Blooms and Depth of Knowledge. Let's look at an example of how a KUD can be developed for both a primary and an intermediate unit.

## FIRST-GRADE PLACE VALUE (BEGINNING OF THE YEAR)

Sample Common Core standards:

> Count to 30, starting at any number less than 30. In this range, read and write numerals and represent several objects with a written numeral.
>
> Understand that the two digits of a two-digit number represent amounts of tens and ones.

Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones—called a "ten."

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

| Students will Know . . . | Students will Understand that . . . | Students will demonstrate knowledge and understanding through the ability to Do . . . |
|---|---|---|
| **K1:** Definitions:<br>more/less<br>numeral/number<br>object/set<br>**K2:** Each number is one more than the number that comes before it.<br>**K3**: Groups of objects can be represented by a numeral that represents the number of objects.<br>**K4:** The last number counted in a group of objects tells how many objects there are. | **U1:** Our numbers follow a pattern that stays the same in all types of numbers.<br>**U2:** Our numbers are based on groups of tens. Any group of ten in one place value equals one of the next place value.<br>**U3:** A number can be represented different ways, e.g., a numeral or group of objects. | **D1:** Count to 30 orally and count on to 30 from any number less than 30.<br>**D2:** Write numbers to 30.<br>**D3:** Represent several objects with a written numeral.<br>**D4:** Tell how many tens and how many ones are in a two-digit number, beginning with 11–19 and going up to 30.<br>**D5:** Count by tens (work up to 100 based on where students are at the beginning of the year).<br>**D6:** Explain why groups of 10 are important in our number system. |

## FOURTH-GRADE DECIMALS

Sample Common Core standards:

> Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*
>
> Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
>
> Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or < and justify the conclusions (e.g., by using a visual model).

| Students will Know . . . | Students will Understand that . . . | Students will demonstrate knowledge and understanding through the ability to Do . . . |
|---|---|---|
| **K1:** Vocabulary: notation, tenths, hundredths, decimal, decimal point, fractional part, whole.<br>**K2:** Decimal numbers can represent any value in our number system, but some are not exactly equivalent values.<br>**K3:** A decimal point separates the whole from the fractional value in a number. Digits after a decimal point represent a fractional part based on denominators that are powers of 10.<br>**K4:** Decimal notation and place value names<br>**K5:** How to write a fraction as a decimal and a decimal as a fraction.<br>**K6:** How to compare decimals to hundredths.<br>**K7:** Adding zeros to the right of the last digit following a decimal point does not change the value of the number.<br>**K8:** You can write decimals in expanded form. | **U1:** Our numbers follow a pattern that stays the same in all types of numbers.<br>**U2:** Our numbers are based on groups of tens. As place values increase (to the right in a number) they are 10 times the previous place value. As they decrease (to the left in a number) they are 1/10 the previous place value.<br>**U3:** A number can be represented different ways. Every fraction can be represented by an infinite number of equivalent fractions, but each fraction is represented by only one decimal (or an equivalent decimal form; for example, 0.25 0.250)<br>**U4:** Every decimal has a specific value, which can be compared through multiple strategies. | **D1:** Determine equivalence by converting between tenths and hundredths.<br>**D2:** Compare and order decimals to hundredths.<br>**D3:** Make estimates appropriate to a given situation or computation with whole numbers and decimals.<br>**D4:** Explain how decimals can represent fractions and how fractions can represent decimals.<br>**D5:** Write a fraction as a decimal and a decimal as a fraction.<br>**D6:** Represent decimal values in multiple representations. |

## WATCH IT!

As you watch Video 3.1, *Planning a Unit Based on Rigorous Mathematical Content,* consider the following questions:

1. How do the teachers make sense of the standards in their unit?
2. What is the difference among Knowing, Understanding, and Doing mathematics?
3. In what ways could the unpacking of the standards influence what the teachers ultimately want students to be able to do, as well as the choice of specific activities in a given lesson?
4. How does writing a KUD for a unit help to align and connect mathematical content vertically (through the grades) as well as from unit to unit?

**Video 3.1** Planning a Unit Based on Rigorous Mathematical Content

Did you notice that all of the understandings from the first-grade unit were also used in the fourth-grade unit, although the topics are seemingly very different? The understanding language will grow up over the years, but the meaning remains the same. This is one hallmark of an understanding. For example, "Our number system is based on groups of ten" is an understanding about a number that will *always* remain true, no matter what the topic, grade level, or skill that is being addressed. If this same idea is referred to over and over again, students make the connection with prior knowledge instead of learning the current skill or topic as a *new* thing that they now need to master. This is the power of understanding mathematics. As we learn more and more, the concepts and connections become the building blocks upon which we grow our knowledge and skills. Instead of learning decimals as a completely new topic with new procedures to be memorized, students can logically see that this is the same idea as the place value based on groups of tens that we have been learning since kindergarten! Now the connections are made, the same principles and patterns apply, and we just learn new details with a new number group.

## TRY IT! KUD LIST

Purpose: To practice explicitly expressing what students should Know, Understand, and be able to Do as a result of learning based on the standards for the unit

What are the big ideas or understandings undergirding one of your units? As you look at a unit, what are the K, U, and D? Use Figure 3.1 as a source for additional understandings.

See the reproducibles available at http://resources.corwin.com/everymathlearnerK-5 for additional understandings, as well as a complete unit template that includes KUD.

Most mathematics resources provide the standards being addressed, and many give “essential questions” for each lesson. Some of these are helpful and others, in and of themselves, are often insufficient. For most teachers, developing KUDs for their unit is the most challenging part of differentiation. Nevertheless, it is worth the effort. The depth at which we come to know our content through this process has many benefits:

- We see connections among mathematics more readily
- We are ready to answer unexpected questions (see Chapter 4)
- We plan purposeful and targeted lessons (see Chapter 4)
- We recognize conceptual gaps and misconceptions in our students more readily (more in Chapter 7)
- We build cohesive units that ensure instruction, tasks, and all forms of assessment reach the desired learning outcomes (more in Chapters 4 and 7)
- Differentiation based on anything less may not target essential learning for all learners and, thus, not have the intended results

### TEACHING UP

One misconception about differentiation that I have often heard is that differentiation “dummies down” curriculum. Nothing could be further from the truth. Research clearly shows that everyone can learn mathematics at high levels. This is the essence of teaching up. We believe that all students can learn, we can hold all students to high expectations, and we must provide the necessary support for students to accomplish the goal. This is fully developed through clarity of curriculum and expectations, instructional decisions (see Chapter 4), and our classroom culture (see Chapter 5).

The discussion of how mathematics should be taught has been going on for a long time. The National Council of Teachers of Mathematics (NCTM) first formally proposed what teaching and learning mathematics might look like in 1989 through the publication of *Curriculum and Evaluation Standards for School Mathematics*. This was followed by the *Professional Standards for Teaching Mathematics* in 1991 and the *Assessment Standards for School Mathematics* in 1995. In 2000, NCTM updated these publications with *Principles and Standards for School Mathematics*. These publications and others began a serious conversation about what it means to learn mathematics. The learning of mathematics in the mathematical community has never been about memorization and speed. In 2001, the National Research Council suggested five strands for mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. All of these publications and resulting conversations and research have laid the foundation for what teaching and learning in mathematics should be ideally—balancing conceptual understandings and reasoning with procedural skills and strategies, embedded within real-world contexts.

Most recently we have NCTM's publication of *Principles to Actions* (2014) describing today's mathematics classroom. The first three principles for school mathematics according to NCTM's *Principles to Actions* are as follows:

- Teaching and Learning—Effective teaching should engage students in meaningful learning that stimulates making sense of mathematical ideas and reasoning mathematically.
- Access and Equity—All students have access to a high-quality mathematics curriculum with high expectations and the support and resources to maximize learning potential.
- Curriculum—A curriculum that develops important mathematics along coherent progressions and develops connections among areas of mathematical study and between mathematics and the real world.

So how do we do it? How do we teach and design units and lessons to accomplish all of this? How do we "teach up" to ensure a high-quality mathematics education for all students? The beginning is certainly clarifying the understandings and basing units and instruction around conceptual understandings with embedded skills. While digging into our standards, we need to make sure that we are teaching at or above our grade-level expectations. Chapter 4 will provide more specific design strategies for students who need some supportive help as well as enrichment ideas.

Teaching up also involves designing lessons, asking questions, and choosing tasks at a high level of cognitive demand. Two structures are commonly used to determine whether we are conducting class at a high level: Depth of Knowledge (DOK) and Cognitive Demand. Both structures have four levels, two levels defined as lower and two defined as upper. DOK is a structure designed by Norman Webb in the late 1990s, and it was originally designed for mathematics and science standards but has been expanded for all content areas. Cognitive Demand (Smith & Stein, 1998) is a structure specifically for mathematics. Figure 3.2 gives the level names for each framework and their characteristics.

**FIGURE 3.2**

### STRUCTURES FOR COGNITIVE COMPLEXITY

| Level | Depth of Knowledge | Cognitive Demand | Description |
|---|---|---|---|
| Lower Level 1 | Recall | Memorization | • Reproducing facts, rules, formulas, procedures, or definitions from memory.<br>• No connection to concepts. |
| Lower Level 2 | Skill/Concept | Procedures Without Connections | • Uses information in a familiar situation.<br>• Involves two or more steps.<br>• Algorithmic. Use a procedure rotely.<br>• Very little ambiguity or reasoning involved.<br>• No student explanations required. |

| Level | Depth of Knowledge | Cognitive Demand | Description |
|---|---|---|---|
| Upper Level 3 | Strategic Thinking | Procedures With Connections | • Requires reasoning, developing a plan, or defining a sequence of steps.<br>• Some complexity in the task or question.<br>• Procedures are to develop connections and conceptual understanding.<br>• Multiple representations.<br>• Takes cognitive effort. |
| Upper Level 4 | Extended Thinking | Doing Mathematics | • Requires an investigation, time to think, and processing of multiple conditions.<br>• Requires complex and nonalgorithmic thinking.<br>• Explores and understands the nature of mathematical concepts, processes, and relationships.<br>• Mathematics of the real world. |

## TRY IT! HOW RIGOROUS IS IT?

Purpose: To practice determining the level of rigor in a given task or problem

For each of the following tasks, determine the DOK or Cognitive Demand level. Answers are at the end of the chapter—but don't cheat!

1. Jeff had 64 action figures. He gave 12 figures to his sister. Then he divided the remaining figures equally among his FOUR friends. How many figures did each of his friends get?
2. Identify the place value of the underlined digit:
   a. 368
   b. 252
3. Solve the following problems. Check your answers with a calculator:
   a. 24 × 13
   b. 832 ÷ 4

4. Survey your classmates to find out how many brothers, sisters, dogs, and cats they have. Make a bar chart of one of the choices (brothers, sisters, dogs, and cats). What comparisons can you make? What questions can you ask? Find a partner who graphed a different choice. What comparisons can you make? What questions can you ask?

5. Use Base 10 blocks to model regrouping with addition and subtraction. Solve the following problems using Base 10 blocks:
   a. 42 – 17
   b. 38 + 27

Most mathematics instruction in the United States is at levels 1 and 2 only. Level one should be a supporting level to enable students to function at levels 2 and 3. Please understand that these levels are not hierarchical—that is, you do not move through the levels in order! In fact, starting with a level 4 task is often a great starting point to create the need to learn the facts, formulas, procedures, and other skills in levels 1 and 2. We need to aim for the majority of our work to be at levels 2 and 3. In Chapter 4, we will further discuss the selection and design of tasks.

There is yet another consideration to teaching up: helping students understand what it means and looks like to be an active learner of mathematics.

**Consider It!**

Make a list of the tasks and questions you have used in class over the past week or two. Write the cognitive level next to each one. At what level are most of the tasks and questions with which your students engage? Do you need to make any adjustments?

## WHAT LEARNING MATHEMATICS WITH UNDERSTANDING LOOKS LIKE

*How* students engage with learning mathematics is equally as important as the content they are learning. In fact, if they are not invested in the process of learning, they may not learn the content to the depth we desire. As mentioned, this is not a new way of thinking, but it has rarely been made an integral part of a standards document or mathematics learning.

The realization of how important it is that students develop “mathematical habits of mind” has prompted our current focus on describing and expecting that the way students learn mathematics shifts along with the content of what students are learning. Today’s standards

**Consider It!**

What does it look like when students are involved with learning mathematics? What verbs come to mind?

documents describe student actions for learning in various ways. The Common Core State Standards have described them through the Standards for Mathematical Practice. Other states that have not adopted the Common Core also have Process standards that are very similar, and some states that are not using the Common Core State Standards are using the Standards for Mathematical Practice as part of their state's standards document. These behaviors are written as standards to raise the importance of students' actions and thinking in learning mathematics effectively, and they are not only expected in the classroom but also are expected to be assessed as a part of end-of-year testing. Although there are slight differences in the descriptions of each process, Figure 3.3 shows alignment among the Standards for Mathematical Practice with other states' process standards or goals.

## WATCH IT!

Amy Francis taught her second-grade students the 8 Standards for Mathematical Practice at the beginning of the school year. As you watch Video 3.2, *Putting the Standards for Mathematical Practice at the Heart of Differentiation*, consider the following questions:

1. What do you believe Mrs. Francis did to establish how students participate in and engage with mathematical content in deep and meaningful ways?
2. In what ways do you see evidence that the students are aware of their own, and others', learning process?
3. What are the pros and cons in the students' actions in learning mathematics in this way?
4. Why do you believe there is such an emphasis on mathematical practices (or processes or habits of mind) in the learning of mathematics today?

**Video 3.2** Putting the Standards for Mathematical Practice at the Heart of Differentiation

Although the various documents may give different names, there is agreement on what learning mathematics should look like. Combined into a simplified list, Figure 3.3 compares the Standards for Mathematical Practice with other states' standards. Remember that these are describing student actions, not teacher actions! I believe we all do these things as mathematics teachers. In fact, if we want our students to exhibit these behaviors, we do need to model them, as well as to teach and expect them from our students.

FIGURE 3.3

## STANDARDS FOR MATHEMATICAL PRACTICE (SMP) AND PROCESS STANDARDS[1]

| SMP | NE | OK | SC | TX | VA |
|---|---|---|---|---|---|
| Make sense of problems and persevere in solving them | Solves mathematical Problems | Develop a deep and flexible conceptual understanding | Make sense of problems and persevere in solving them | Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution | Mathematical Problem Solving |
| Reason abstractly and quantitatively |  | Develop mathematical reasoning | Reason both contextually and abstractly |  | Mathematical Reasoning |
| Construct viable arguments and critique the reasoning of others | Communicates mathematical ideas effectively | Develop the ability to communicate mathematically | Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others | Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate | Mathematical communication |
| Model with mathematics | Models and represents mathematical problems | Develop the ability to conjecture, model and generalize | Connect mathematical ideas and real-world situations through modeling | Create and use representations to organize, record, and communicate mathematical ideas | Mathematical Representations |

| SMP | NE | OK | SC | TX | VA |
|---|---|---|---|---|---|
| Use appropriate tools strategically | | Develop strategies for problem solving | Use a variety of mathematical tools effectively and strategically | Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental mathematics, estimation, and number sense as appropriate, to solve problems | Mathematical Problem Solving |
| Attend to precision. | | Develop accurate and appropriate procedural fluency | Communicate mathematically and approach mathematical situations with precision | Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication | Mathematical Problem Solving |
| Look for and make use of structure | | Develop the ability to conjecture, model and generalize | Identify and utilize structure and patterns | Analyze mathematical relationships to connect and communicate mathematical ideas | |
| Look for and express regularity in repeated reasoning | | Develop the ability to conjecture, model and generalize | Identify and utilize structure and patterns | | |
| | Makes mathematical connections | Develop a productive mathematical disposition | | Apply mathematics to problems arising in everyday life, society, and the workplace | Mathematical Connections |

## Consider It!

As you read through the combined list of mathematical learning practices, create a plan for how you will teach your students these learning actions.

*Make sense of problems*—reason and interpret mathematical situations. This is the obvious beginning, right? But how often do your students barely look at a problem before saying, “I don’t get it.” We often read the problem to the students, interpret the problem for them, and give the first step. No wonder they do not know how to make sense of problems for themselves. When students engage in learning mathematics, they wrestle with the context of a problem, what they are looking for, possible ways to start the problem, and multiple solution paths or representations. Additionally, they can begin to discuss the mathematics they see in a real-world situation, and they can describe a real-world situation that would require the mathematics being learned.

## TRY IT! HOW STUDENTS MAKE SENSE OF PROBLEMS

Purpose: To shift “sense making” to students when facing a new problem

Give the students a rich problem to solve.

1. In elbow, partners have the first partner read the problem and the second partner interpret the problem in his or her own words. This should also include what the solution will look like (for example, ___ feet). Discuss as a class or check in on students’ interpretations.
2. Have partner 2 suggest a way to start the problem to partner 1. Have partner 1 suggest another way to start or agree to the idea of partner 2 and explain why it will work.
3. Have partners generate strategies to represent and solve the problem.

*Communicate mathematically*—Students explain their thinking mathematically and ask questions of or build on other students’ explanations. Students will use correct mathematical vocabulary and multiple representations to communicate their thinking. Beware of accepting an answer or retelling of steps as an explanation. Explanations need to include reasoning and the meanings and properties of operations to be considered robust.

## TRY IT! STUDENT DISCOURSE

Purpose: To teach healthy mathematical discourse skills

1. Direct mathematical conversations so that they are among the students as much as possible. Do not interpret and redirect questions and answers. Teach students to restate what other students have said. To provide structures for discourse, give students sentence starters such as the following:
    - I agree with ___ because ____
    - Another way to think about this is _______
    - I did it a different way. I ________
    - I disagree with ___ because ____
    - I would like to add on to what ___ said about ____
    - Can you explain what you mean by ____
    - Can you show ____ in another way
    - I think that ________ because ________
2. Ask questions to help students clarify their thinking if they have responded incorrectly rather than calling on another student.
3. Gently correct and provide correct mathematical vocabulary.

*Model with mathematics*—There are two aspects to modeling mathematics: Models of mathematics and how mathematics models the real world. Models of mathematics include using manipulatives such as Base 10 blocks and two-color counters, drawings, and symbols. Whenever new material is presented in a way that students see relationships, they generate greater brain cell activity and achieve more successful long-term memory storage and retrieval (Willis, 2006, p. 15). We also use mathematics to model the real world. When we solve quantitative problems from the world around us, we are modeling the world with mathematics.

## TRY IT! MATHEMATICAL MODELING

Purpose: To make mathematical processes and problems concrete and visual whenever possible.

Challenge students to represent any contextual or numerical problem as many ways as possible. Keep in mind the following tips when using models:

1. The concrete or visual model needs to come before the paper-and-pencil process. If the algorithm is taught first, students will not value or want to complete the concrete or visual activity. Also, the concrete or visual task is to develop the conceptual understanding and make sense of the process or algorithm to follow.
2. Connect the concrete or visual explicitly to the skill or process. If it cannot be explicitly connected, it is not a valid model.
3. Challenge students to represent problems in as many different ways as they can.

*Choose and use tools appropriately*—Tools can be anything! We usually think of physical tools such as number lines, hundreds charts, rulers, counters, and Base 10 blocks. Nevertheless, tools can also be mental strategies such as addition strategies or the distributive property (not by name) for multiplying two-digit numbers.

Knowing which tool or strategy will be appropriate and useful in a given situation is a necessary skill for solving problems and a practical life skill. Often we hand out the tools we will be using in a lesson; for example, today we need rulers. Instead, build a toolbox with all of the mathematical tools in the classroom for table groups or an area where all tools are stored.

## TRY IT! THE TOOLBOX

Purpose: Have students select and defend a variety of mathematical tools

1. For a lesson, explain to the students what the task will be and ask them to choose the tools they think they will need.
2. After the task, ask students to reflect on the tools they chose. Did they get what they needed? Why or why not? Did they choose extra tools that were not needed? Why was that selected?

*Recognize and use patterns and structures*—The more students work with mathematics, the more they can recognize mathematical structures. For example, our whole numbers (and integers in the future) alternate between evens and odds. Place value is a key structure in the elementary grades, and that is why so many standards talk about adding and subtracting 10 or 100 to any number mentally. Structure includes understanding why the process of converting an improper fraction into a mixed number works. Patterns are more than the repeating patterns that are often taught in kindergarten: AB or ABC, and so forth. Recognizing patterns is often more related to repeated reasoning than to labeling how often a color recurs. For example, recognizing a multiplication pattern to determine equivalent fractions would be using repeated reasoning. Noticing that multiplication of whole numbers results in a product greater than the factors but multiplication of a whole number and a fraction less than one results in a product less than the whole factor is using repeated reasoning. Instead of giving students an algorithm, try modeling thinking about the structures and patterns that are inherent in operations and multiple problem examples.

## TRY IT! PATTERN HUNT

Purpose: To make sense of mathematical rules or procedures, and to recognize mathematical structures that give hints to solutions

1. Give students several problems to solve that are solved using manipulatives, drawings, models, and so on, but not rules or steps.
2. Generate a list of the problems and answers.
3. Have the students find the "short cuts" or patterns they recognize. For example, multiplication facts of 5 will show all products end in either 5 or 0. Multiplication of whole numbers results in a product greater than the factors, but multiplication with a fraction less than one will result in a product less than at least one factor.
4. This will undoubtedly be the algorithm you wanted to teach, and instead it will be a student discovery.

*Attend to precision*—Certainly undergirding all other of these mathematical practices is the ability to attend to precision. Precision includes using correct vocabulary. It includes noticing whether an equation has a plus or minus sign and all other notation. It includes knowing mathematical facts and efficiently using various strategies for operations. It includes knowing when and how to apply the properties of operations.

## TRY IT! CATCH ME

Purpose: Have students catch you any time you are not mathematically precise

1. Prepare a problem presentation with which you will make precision errors.
2. Use incorrect or slang vocabulary. Make arithmetic mistakes.
3. Have students find your imprecisions.
4. You can also divide the class into two teams, and award points as students collaborate to find the errors.

Clarifying content and teaching children how to learn mathematics actively is essential for all mathematics instruction. It is certainly necessary for effective differentiation, which is based on solid curriculum.

## CONCLUSION

We have developed a complete picture of clarifying content for designing differentiated instruction. We have also discussed the actions students need to employ to learn mathematics with understanding. Effective teaching is not about delivering information or creating meaning. It's melding the two to help students see the meaning in the information they are learning (Sousa & Tomlinson, 2011).

As we prepare to differentiate our mathematics instruction, consider the changes in how we teach mathematics as a whole. Figure 3.4 compares before and after of teaching mathematics, adapted from David Sousa (2015).

FIGURE 3.4

### BEFORE AND AFTER OF MATHEMATICAL REASONING

| We used to teach mathematics as . . . | But now we teach mathematics as . . . |
|---|---|
| Problems to be calculated | Situations about which we reason |
| Procedures to be memorized | Operations that are based on properties with multiple representations and strategies |
| Isolated topics | Connected concepts |
| A speed activity for prowess | Problem solving and reasoning for prowess |
| Teacher-led and valued | Student-discovered and valued |
| Something forgettable | Understood, so remembered |

There are two keys to differentiation: Know your content, and know your students. In the last chapter, we looked at strategies to know our students as learners. In this chapter, we looked at how to know our content. In the next chapter, we will look at how knowing our content and our students come together in powerful differentiation.

## FREQUENTLY ASKED QUESTIONS

Q: With the Common Core standards and other state standards so closely aligned, do we really need to go through the work of writing a KUD? Aren't they written somewhere?

A: There are many posts about big ideas online. Some are good and can be a resource. Nevertheless, some are labeled "conceptual understanding" but are actually fact or skill based. These are not understandings. Additionally, there is nothing like the struggle to make sense of the standards to help your own learning and clarify what you want students to come away with. Remember that we want our students to take a challenge and struggle with things that are challenging; therefore, we need to do the same.

Q: What if my students can't explain their thinking?

A: Chances are pretty good that your students have been asked to tell how they got an answer in mathematics, and this has always been what an "explanation" was. They need to be taught how to construct a mathematical explanation. This can be done by modeling first and foremost but also by asking questions such as "how did you know to do that" or "what allows you to do that in math (e.g., you can add in any order, etc.)."

Q: What about students who can't reach the standard?

A: It is very important to teach the grade-level standards. When we draw conclusions that certain students cannot reach the standard and, therefore, lower the expectations or, worse, lower the instruction level, we widen gaps not close them. Truthfully, most students can reach the standards given support and, if appropriate, more direct intervention. The RTI structure is designed to enable students significantly behind in learning to close gaps and reach as close to grade level as possible, if not actually reach the standard. Yet even given the RTI structure, remember that Tier 1 instruction is on grade level. Chapter 4 will more specifically address how to design for readiness.

# Keepsakes and Plans

What are the keepsake ideas from this chapter, those thoughts or ideas that resonated with you that you do not want to forget?

Mathematics Makes Sense:

1.

2.

3.

Analyzing Standards and Developing Conceptual Understandings in Mathematics:

1.

2.

3.

Mathematical Learning Actions:

1.

2.

3.

Teaching Up:

1.

2.

3.

Based on my keepsake ideas, I plan to:

1.

2.

## TRY IT! ANSWERS

1. DOK 2/Procedures without connections. This item is an application of computational algorithms. It is a multistep problem requiring the student to make a decision on how to approach the computations.
2. DOK 1/Memorization. This is straight memorization of place value names.
3. DOK 2/Procedures without connections. This is another multistep procedure without any connections, sense-making, or application.
4. DOK 4/Doing Mathematics. This is an example of what mathematicians do in the real world.
5. DOK 3/Procedures with Connections. This task uses models to make sense of the algorithms that will be used with regrouping, and reinforcing groups of tens and place value.

## NOTE

1. The full descriptions of each of these mathematical behaviors can be accessed at the following websites:
    - Common Core Standards for Mathematical Practices: http://www.corestandards.org/Math/Practice/
    - Nebraska Mathematical Processes: https://www.education.ne.gov/math/Math_Standards/Adopted_2015_Math_Standards/2015_Nebraska_College_and_Career_Standards_for_Mathematics_Vertical.pdf
    - Oklahoma Mathematical Actions and Processes: http://sde.ok.gov/sde/sites/ok.gov.sde/files/documents/files/OAS-Math-Final%20Version_2.pdf
    - South Carolina Process Standards: https://ed.sc.gov/scdoe/assets/file/agency/scde-grant-opportunities/documents/SCCCRStandardsForMathematicsFinal-PrintOneSide.pdf
    - Texas Process Standards: http://www.abileneisd.org/cms/lib2/TX01001461/Centricity/Domain/1943/Texas%20Mathematical%20Process%20Standards%20Aug%202014.pdf
    - Virginia Standards of Learning Mathematics Goals: http://www.pen.k12.va.us/testing/sol/standards_docs/mathematics/index.shtml

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