Part 3

Number and Operations—Fractions

Number and Operations—Fractions

Domain Overview

GRADE 3

Students use visual models, including area models, fraction strips, and the number line, to develop conceptual understanding of the meaning of a fraction as a number in relationship to a defined whole. They work with unit fractions to understand the meaning of the numerator and denominator. They build equivalent fractions and use a variety of strategies to compare fractions. In Grade 3, denominators are limited to 2, 3, 4, 6, and 8.

GRADE 4

Fourth graders extend understanding from third grade experiences, composing fractions from unit fractions and decomposing fractions into unit fractions, and apply this understanding to add and subtract fractions with like denominators. They begin with visual models and progress to making generalizations for addition and subtraction fractions with like denominators. They compare fractions that refer to the same whole using a variety of strategies. Using visual models and making connections to whole number multiplication supports students as they begin to explore multiplying a whole number times a fraction. In Grade 4, denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100. Students build equivalent fractions with denominators of 10 and 100 and connect that work to decimal notation for tenths and hundredths.

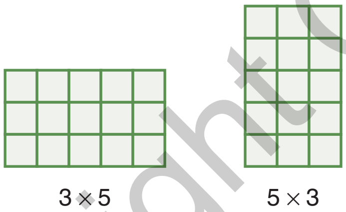
GRADE 5

Fifth graders build on previous experiences with fractions and use a variety of visual models and strategies to add and subtract fractions and mixed numbers with unlike denominators. Problem solving provides contexts for students to use mathematical reasoning to determine whether their answers make sense. They extend their understanding of fractions as parts of a whole to interpret a fraction as a division representation of the numerator divided by the denominator. Students use this understanding in the context of dividing whole numbers with an answer in the form of a fraction or mixed number. They continue to build conceptual understanding of multiplication of fractions using visual models and connecting the meaning to the meaning of multiplication of whole numbers. The meaning of the operation is the same; however, the procedure is different. Students use visual models and problem solving contexts to develop understanding of dividing a unit fraction by a whole number and a whole number by a unit fraction. Once conceptual understanding is established, students generalize efficient procedures for multiplying and dividing fractions.

SUGGESTED MATERIALS FOR THIS DOMAIN

3	4	5	
	✓	✓	Decimal models (base-ten blocks) (Reproducible 4)
✓	✓	✓	Fraction area models (circular) (Reproducible 5)
✓	✓	✓	Fraction area models (rectangular) (Reproducible 6)
✓	✓	✓	Fraction strips/bars (Reproducible 7)
✓	✓	✓	Grid paper (Reproducible 3)
✓	✓	✓	Objects for counting, such as beans, linking cubes, two-color counter chips, coins
✓	✓	✓	Place value chart (Reproducible 8)

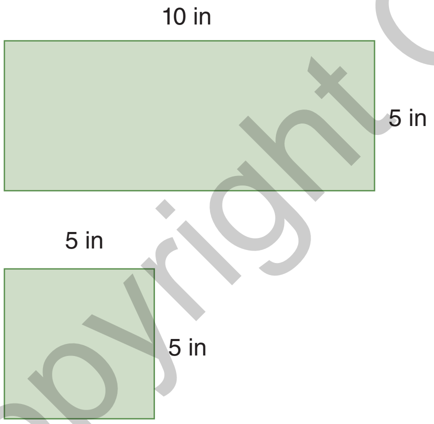
KEY VOCABULARY

3	4	5	
✓	✓	✓	**area model** a concrete model for multiplication or division made up of a rectangle. The length and width represent the factors, and the area represents the product. 
✓	✓	✓	**benchmark** a number or numbers that help to estimate or determine the reasonableness of an answer. Sample benchmarks for fractions include 0, $\frac{1}{2}$, 1.
	✓	✓	**decimal fraction** a fraction whose denominator is a power of 10, written in decimal form (for example, 0.4, 0.67)
✓	✓	✓	**denominator** the number of equal-sized pieces in a whole, the number of members of a set with an identified attribute. The bottom number in a fraction.
	✓	✓	**equivalent fractions** fractions that name the same amount or number but look different (*Example:* $\frac{2}{3}$ and $\frac{6}{9}$ are equivalent fractions)
	✓	✓	**hundredth** one part when a whole is divided into 100 equal parts
	✓	✓	**like denominator (common denominator)** having the same denominator
	✓	✓	**like numerator (common numerator)** having the same numerator

(Continued)

(Continued)

KEY VOCABULARY

3	4	5	
✓	✓	✓	**measurement division (equal groups model)** a division model in which the total number of items and the number of items in each group is known. The number of groups that can be made is the unknown. *Example:* I have 3 yards of ribbon. It takes $\frac{1}{6}$ of a yard to make a bow. How many bows can I make? (How many groups of $\frac{1}{6}$ yards can I make from 3 yards?)
	✓	✓	**mixed number** a number that is made up of a whole number and a fraction (for example, $2\frac{3}{4}$)
✓	✓	✓	**numerator** the number in a fraction that indicates the number of parts of the whole that are being considered. The top number in a fraction.
✓	✓	✓	**partitive division (fair share model)** a division model in which the total number and the number of groups is known and the number of items in each group is unknown. *Example:* Erik has $\frac{1}{2}$ of a gallon of lemonade. He wants to pour the same amount in 5 glasses. How much lemonade will he pour into each glass if he uses all of the lemonade?
		✓	**scale (multiplication)** compare the size of a product to the size of one factor on the basis of the size of the other factor *Example:* Compare the area of these rectangles. When you double *one* dimension, the area is doubled. 
	✓	✓	**tenth** one part when one whole is divided into 10 equal parts
✓	✓	✓	**unit fraction** a fraction with a numerator of one, showing one of equal-sized parts in a whole (for example, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$)

Number and Operations—Fractions[1]

3.NF.A*

[1] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Develop understanding of fractions as numbers.

STANDARD 1

3.NF.A.1: Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

STANDARD 2

3.NF.A.2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

STANDARD 3

3.NF.A.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

Examples: Express 3 in the form $3 = \frac{3}{1}$; *recognize that* $\frac{6}{1} = 6$; *locate* $\frac{4}{4}$ *and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

*Major cluster

Number and Operations—Fractions[1] 3.NF.A

Cluster A: Develop understanding of fractions as numbers.

[1] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Grade 3 Overview

As students begin to develop understanding of fractions as a special group of numbers, they work with area models (circles, rectangles, and squares), fraction strips and fraction bars, and the number line to explore the meaning of the denominator and the meaning of the numerator. Unit fractions, fractions with a numerator of 1, form the foundation for initial fraction work. Students extend work with unit fractions to comparing fractions and finding simple equivalent fractions. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8, which provides an opportunity to develop deep understanding of these foundational concepts.

Standards for Mathematical Practice
SFMP 2. Use quantitative reasoning.
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 5. Use appropriate tools strategically.
SFMP 6. Attend to precision.
SFMP 7. Look for and make use of structure.
SFMP 8. Look for and express regularity in repeated reasoning.

As third graders begin formal work with fractions, first and foremost they understand that fractions are numbers. They reason with physical models including area models, fraction strips, and number lines to understand unit fractions, such as $\frac{1}{4}$, as one part of a defined whole cut into four equivalent parts. They begin to develop an understanding of the meaning of the numerator and the denominator. Students extend their understanding of the structure of fractions beyond unit fractions, using visual representations to explain their thinking. They use repeated reasoning to compose other fractions from unit fractions including fractions equal to or greater than 1. Connecting area models to fraction strip models and to number lines provides a meaningful progression of models. This helps students to make generalizations as they build understanding of the meaning of common fractions extended to fractions greater than one. They use this understanding to compare and find equivalent fractions.

Related Content Standards

1.G.A.3 2.G.A.3 3.G.A.2 4.NF.A.1 4.NF.A.2 4.NF.B.3 4.NF.C.5

Notes

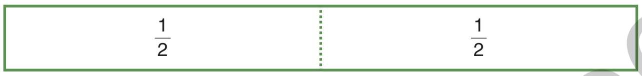
STANDARD 1 (3.NF.A.1)

Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

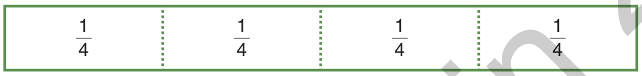
Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

A fundamental goal throughout work across fraction clusters is for students to understand that fractions are numbers. They represent a quantity or amount that happens to be less than, equal to, or greater than 1. Too often we project the notion of fractions as parts of a whole without emphasizing that they are special numbers that allow us to count pieces that are part of a whole. Fractions in third grade are about a whole being divided (partitioned) into equal parts. Suggested models for Grade 3 include area models (circles, squares, rectangles), strip or fraction bar models, and number line models. Set models (parts of a group) are not models used in Grade 3. This Standard is about understanding unit fractions (fractions with a numerator of 1) and how other fractions are composed of unit fractions.

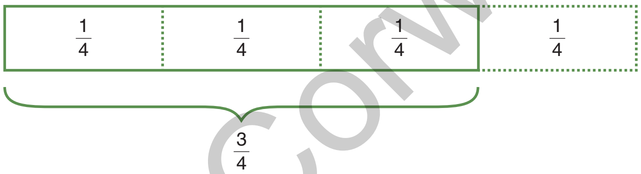
Folding a strip into 2 equal parts (one fold), each part or section would be $\frac{1}{2}$.



Folding a strip into 4 equal parts (three folds), each part or section would be $\frac{1}{4}$.

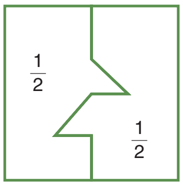


The fraction $\frac{3}{4}$ is the quantity formed by 3 parts that are each $\frac{1}{4}$ of the whole.

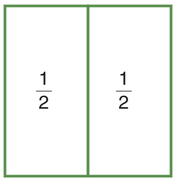


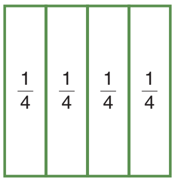
Important ideas for students to consider as they begin their work with fractional parts include:

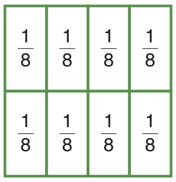
- When working with any type of area model (circles, squares, rectangles) or strip models, fractional parts must be of equal size (but not necessarily equal shape). Using grid paper or geoboards can help students to determine when two pieces are the same size even if they are not the same shape.



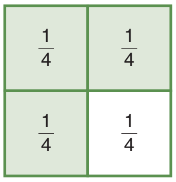
- The denominator represents the number of equal size parts that make a whole.
- The more equal pieces in the whole (greater denominator), the smaller the size of the piece.







- The numerator of a fraction represents the number of equal pieces in the whole that are counted.



$\frac{3}{4}$ are shaded.

What the TEACHER does:

- Begin with strip models. These can simply be strips of construction paper about 2 inches by 11 inches. It is important that students understand that one strip represents one whole. If it is possible to use different colors it will help students to identify and compare fractions.
- Have students fold one strip into 2 equal parts and label each part $\frac{1}{2}$.
    - ○ Ask students to make a conjecture about the meaning of the 2 in $\frac{1}{2}$ (the number of equal-size parts the whole strip).
    - ○ Ask students to make a conjecture of the meaning of the 1 in $\frac{1}{2}$ (each piece is one part of the whole).
- Repeat the process folding and labeling strips for fourths, eighths, thirds, and sixths.
- Introduce the terms *numerator* and *denominator*. Ask students to explain what each term means based on this activity.
- Show students $\frac{3}{4}$ of a strip. Ask them what part (fraction) of one whole strip that amount represents. Students should use the terminology *numerator* and *denominator* in justifying their reasoning (that is, I know it is $\frac{3}{4}$ because it is made up of $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$).
- Prepare other activities in which students name parts of a whole and describe them as the sum of unit fractions.
- Use a variety of concrete representations for activities in which students compare the size of various unit fractions and then develop an understanding that the larger the denominator, the smaller the size of the piece. Using the same size whole is an important part of this understanding.
- When students are ready, use two different size wholes to have them talk about when $\frac{1}{4}$ might be greater than $\frac{1}{2}$. (When $\frac{1}{4}$ is part of a larger whole than $\frac{1}{2}$.)
- Give examples of fraction models that are equal size but not equal shape. Use area models or geoboards to have students represent unit fractions that are equal sized but not equal shape.

What the STUDENTS do:

- Make models of fractions (with denominators of 2, 3, 4, 6, and 8) using fraction strips. Label each part with the correct unit fraction.
- Describe the meaning of the denominator and the numerator using pictures, numbers, and words.
- Name various parts of the whole using fractions and explain that the fraction is made up of that number of unit pieces.
  $\frac{5}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$
- Demonstrate an understanding that given the same size whole, the larger the denominator the smaller the size of the pieces because there are more pieces in the whole. Students demonstrate understanding by explaining their reasoning using concrete materials, pictures, numbers, and words.
- Identify and demonstrate fractional parts of a whole that are the same size but not the same shape using concrete materials.

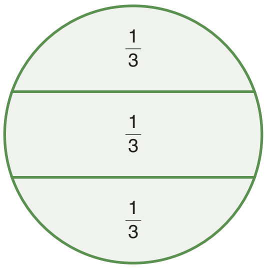
Addressing Student Misconceptions and Common Errors

There are many foundational fraction ideas in this Standard, and it is important to take the time necessary to develop student understanding of each idea. This is best accomplished through extensive use of concrete representations, including fraction strips, area models, fraction bars, geoboards, and similar items. Do not work with too many representations at the same time. Begin with activities that use area models and reinforce those idea with fraction strips and then number lines. For most students one experience with a concept will not be adequate to develop deep understanding.

Students who demonstrate any of the following misconceptions need additional experiences connecting concrete representations to fraction concepts:

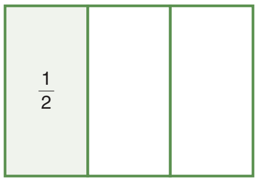
- Given the same size whole, the smaller the denominator, the smaller the piece.
- Fraction pieces must be the same shape and size.

- Students write a fraction numeral based on the number of pieces in a whole even if they are not the same sized pieces.



Misconception: Student considers the number of pieces in the whole but does not understand they must be the same size.

- Student label fractions as $\frac{\text{part}}{\text{part}}$ rather than as $\frac{\text{part}}{\text{whole}}$.



Misconception: Student writes the fraction as a part to part relationship rather than $\frac{1}{3}$ (part to whole).

Notes

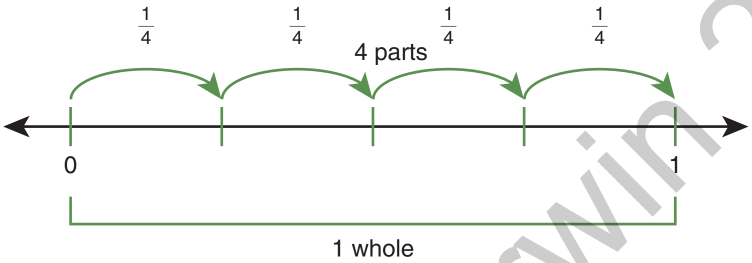
STANDARD 2 (3.NF.A.2)

Understand a fraction as a number on the number line; represent fractions on a number line diagram.

Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

a. *Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.*

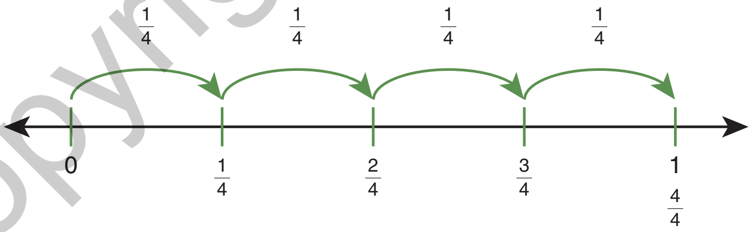
Students have had previous experience with whole numbers on the number line. They extend this understanding by focusing on subdividing the distance from 0 to 1. Representing $\frac{1}{4}$ on the number line requires students to understand the distance from 0 to 1 represents one whole. When they partition this distance, the whole, into 4 equal parts, each part has the size of $\frac{1}{4}$. They also reason and justify the location of unit fractions by folding strips or on the number line. Previous work with fraction strips or fraction bars can be extended to developing parts on the number line.



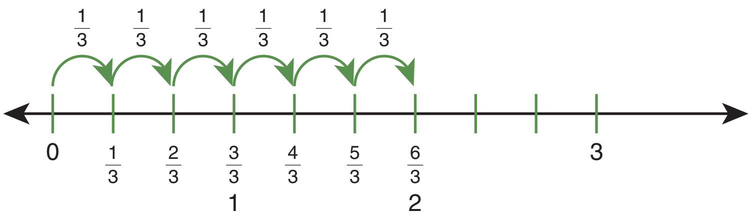
b. *Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a length $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.*

As students develop conceptual understanding of unit fractions they extend this to work counting unit fractions to represent and name other fractions on the number line.

For example, represent the fraction $\frac{3}{4}$ on a number line by marking off lengths of $\frac{1}{4}$ starting at 0. They can explain that 3 pieces of $\frac{1}{4}$ $(\frac{1}{4} + \frac{1}{4} + \frac{1}{4})$ or that the distance from 0 to that point represents $\frac{3}{4}$ on the number line.

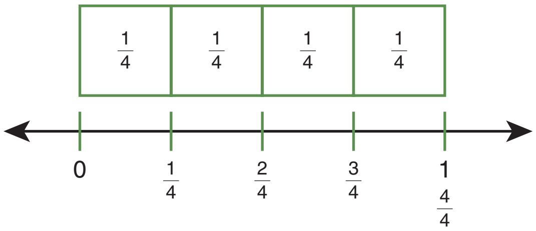


This Standard also includes work with improper fractions, not as a special group of fractions but as a continuation of counting unit fractions. By extending the number line, students develop the understanding that fractions equal to 1 have the same numerator and denominator and fractions greater than 1 have a numerator that will be greater than the denominator. They develop this understanding by counting on the number line using unit factions and recognizing patterns with fractional numbers.



What the TEACHER does:

- Provide students with fraction strips (Reproducible 7) and number lines and ask students to transfer the parts from the fraction strip to the number line.
- Model labeling unit fraction intervals on the number line.
- Ask students to use the unit fraction intervals to "count" and label the fraction name for each division from zero to one.



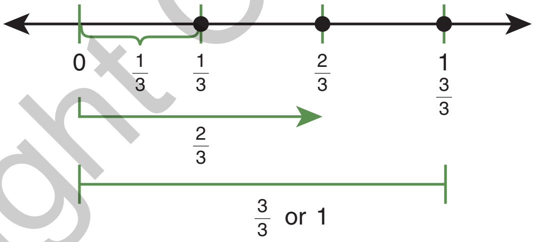
- Facilitate discussions in which students explain their reasoning as they label the number line.
- Repeat this process for fractions with denominators of 2, 3, 4, 6, and 8.
- Extend the number line to numbers greater than 1 using the same rationale for naming points on the number line.
- Provide students with many opportunities to describe patterns they see as they label number lines.

What the STUDENTS do:

- Use fraction strips to find fractional parts on the number line.
- Label intervals and points on the number lines. Intervals are unit fractions. Points on the number line represent the distance from 0 to that specific point and are made up of the number of unit fraction intervals.
- Demonstrate how they labeled the number line and explain their thinking.
- Extend number lines and activities to include fractions greater than 1.

Addressing Student Misconceptions and Common Errors

Although it is not critical for students to differentiate between the intervals between points and actual points on the number line, you want to be careful not to cause any misconceptions. The fraction that names a point on the number line describes the distance of that point from 0 and not the point itself.



Notes

STANDARD 3 (3.NF.A.3)

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

a. *Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.*

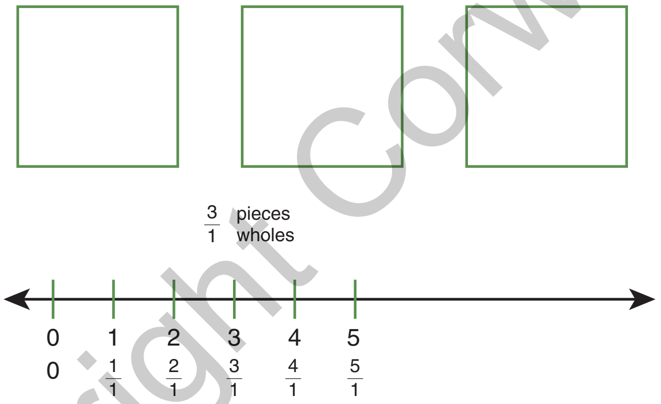
The number line is one of several models such as area models and fraction bar models that can help students to develop conceptual understanding of equivalent fractions. Concrete experiences drawing area models and folding fraction strips should gradually transition to equivalent fractions on the number line.

b. *Recognize and generate simple equivalent fraction, e.g.,* $\frac{1}{2}=\frac{2}{4}, \frac{4}{6}=\frac{2}{3}$. *Explain why the fractions are equivalent, e.g., by using a visual fraction model.*

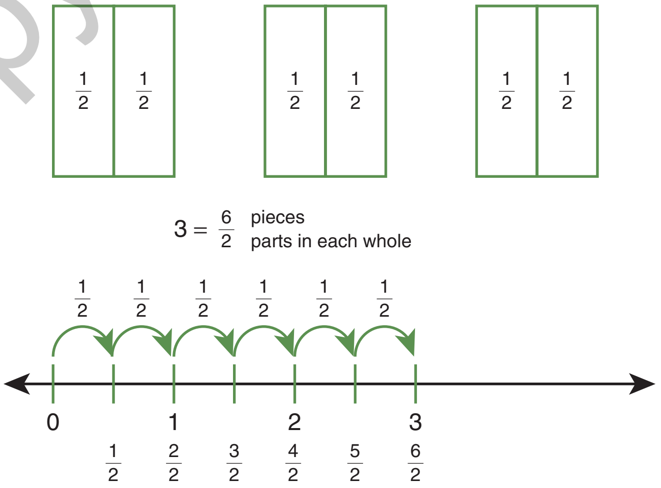
Patterns with visual models help students to reason and justify why two fractions are equivalent. The use of procedures or algorithms is not a third grade expectation.

c. *Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.* Examples: Express 3 in the form $3=\frac{3}{1}$; recognize that $\frac{6}{1}=6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.

The foundational understanding of this Standard is established by providing experiences for students to recognize that any whole number can be expressed as a fraction with a denominator of 1. Previous experiences developing the understanding that the denominator tells the number of pieces into which one whole has been partitioned now extends to situations in which the whole is not divided and remains in 1 piece, resulting in a denominator of 1.



Students extend this understanding to dividing a number of area models that are wholes into parts and determining the resulting fraction.

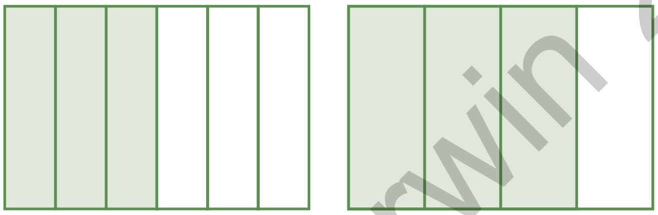


Classroom discussions and visual representations lead students to make the connection between fraction representations and division. For example, the fraction $\frac{6}{2}$ represents 6 pieces that are each $\frac{1}{2}$ of one whole. Two pieces are needed to make one whole. Modeling by putting the wholes back together with each whole representing one group shows that I can make 3 wholes or groups, each of which is $\frac{2}{2}$. Therefore $\frac{6}{2}$ is the same as 6 ÷ 2. Note that students are just beginning to make this connection, and multiple activities will help students to develop this understanding rather than teaching it by simply giving them a rule.

d. *Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.*

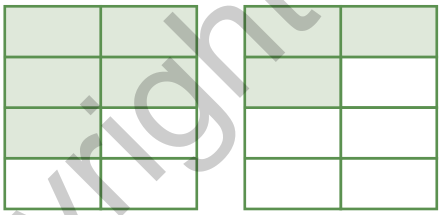
Students work with models and the number line to compare fractions with the same numerator. Models should refer to the same whole and include examples of how different size wholes impact the size of the fraction. Students explain their reasoning using pictures, words, and numbers, focusing on the meaning of the denominator as describing the number of pieces in one whole or size of the pieces and the numerator as the number of pieces or count. If the pieces are the same size (denominator), then the number of pieces (numerator) will determine which fraction is greater.

$\frac{3}{6} < \frac{3}{4}$ because sixths are smaller than fourths



Students extend their reasoning to compare fractions with different denominators and the same numerator using models and the number line, and explain their reasoning using pictures, words, and numbers. They generalize that when the number of pieces (numerator) is the same, the number of pieces in a whole (denominator) will determine which fraction is greater. The larger the denominator, the smaller the size of the piece.

$\frac{5}{8} > \frac{3}{8}$



because 5 pieces that are eighths are more than 3 pieces that are eighths

What the TEACHER does:

- Provide a variety of activities with visual models, including area models, fraction strips, and the number line, to give students experience developing conceptual understanding that
    - ○ Many fractions can describe the same quantity or point on a number line.
    - ○ Fractions that represent the same amount are called *equivalent fractions*.
- Use purposeful questions to help students recognize patterns in equivalent fractions.
    - ○ What do you notice about the numerators in equivalent fractions?
    - ○ What do you notice about the denominators in equivalent fractions?
    - ○ What patterns do you see in the numerators and denominators in two equivalent fractions?
- Connect concrete experiences to building sets of equivalent fractions using numerals. This should not be based on a procedure or algorithm, rather by looking for patterns and having students describe what is happening to the visual representation and numbers as they find equivalent fractions.
- Provide activities and experiences in which students use visual representations to express whole numbers as fractions.

(continued)

What the TEACHER does (continued):

  - ○ Cutting one whole into fourths shows that $\frac{4}{4}$ equals one whole.
  - ○ Generating fractions from more than one whole. Cutting 4 wholes into thirds will result in 12 pieces. Because each piece is $\frac{1}{3}$ of a whole, the resulting fraction is $\frac{12}{3}$. Therefore $\frac{12}{3}$ is equivalent to four wholes.
  - ○ Leaving several wholes intact shows that 4 can be represented as $\frac{4}{1}$ since there are 4 pieces that are each 1 whole piece.
- Provide concrete experiences for students to compare parts of the same size whole with the same numerator and different denominators. Ask questions that will help students to generalize that when the size of the piece (denominator) is the same, the number of pieces (numerator) will determine which is the greater fraction.
- Provide concrete experiences for students to compare fractions of the same size whole with the same denominator and different numerators and generalize that when the number of pieces in the whole is the same (denominator), the number of pieces (numerator) will determine which fraction is greater. The larger the denominator, the smaller the size of the piece.

What the STUDENTS do:

- Use visual representations including rectangular and circular area models, fraction bars, and the number line to find various (equivalent) fractions that name the same quantity or point.
- Build sets of equivalent fractions from visual models and by recognizing patterns.
- Explain their reasoning in building sets of equivalent fractions. For example, $\frac{3}{4}$ is equivalent to $\frac{6}{8}$ because doubling the number of pieces in the whole (denominator) then will also double the count of pieces (numerator).
- Use visual representations to find fractional names for 1.
- Use visual representations to find fractional names for several wholes that are not partitioned (denominator is 1).
- Use visual representations to find fractional names for several wholes that are partitioned into pieces.
- Explain patterns they see as they are working with wholes and their equivalent fractions.
- Provide experiences that help students to make the following generalizations:
  - ○ When the numerator and denominator are the same, the value of the number is one whole.

    $\frac{6}{6} = 1 \qquad 1 = \frac{8}{8} \qquad \frac{4}{4} = 1$
  - ○ When the denominator is 1, the fraction represents wholes. The number of wholes is the same as the numerator.

    $\frac{8}{1} = 8 \qquad 7 = \frac{7}{1} \qquad 3 = \frac{3}{1}$
  - ○ When the numerator is a multiple of the denominator, the number of wholes is their quotient.
  - ○ $\frac{12}{4} = 3 \qquad \frac{10}{2} = 5 \qquad 6 = \frac{18}{3}$

Addressing Student Misconceptions and Common Errors

As students work with equivalent fractions, it is important that they understand that different fractions can name the same quantity and there is a multiplicative relationship between equivalent fractions. Students need multiple experiences using concrete materials as they explore each of these important concepts. They need to explain their reasoning and explicitly connect visual representations (concrete and pictorial) to numerical representations. It is important that students have time to make these connections, describe patterns, and make generalizations rather than by practicing rote rules.

The following misconceptions indicate that students need more work with concrete and then pictorial representations:

- The numerator cannot be greater than the denominator.
- The larger the denominator, the larger the piece.
- Fractions are a part of a whole; therefore, you cannot have a fraction that is greater than 1 whole.
- In building sets of equivalent fractions, students use addition or subtraction to find equivalent fractions.

Notes

Sample PLANNING PAGE

Standard: 3.NF.A.1. *Understand a fraction* $\frac{1}{b}$ *as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction* $\frac{a}{b}$ *as the quantity formed by a parts of size* $\frac{1}{b}$.

Mathematical Practice or Process Standards:

SFMP 4. Model with mathematics.

Students make fraction strips to use as they begin to explore the meaning of fractional parts of a whole.

SFMP 6. Attend to precision.

Initial experiences with fractions emphasize that a fraction is a number. Students develop fraction-related vocabulary, starting with *numerator* and *denominator*.

Goal:

Students use physical models as they begin to work with fractions, focusing on the meaning of fractions as a number as well as the meanings of the numerator and the denominator.

Planning:

Materials: 3 inch by 12 inch construction paper strips for each student. If possible provide each student with five strips that are different colors. Be sure to have extra strips on hand for students who make a mistake. Color marking pens.

Sample Activity:

Begin with one strip. Designate that strip as one whole and label it 1 WHOLE.

Have students take a strip of a color (for example, red) and fold it into two parts that are the same size. Talk about the pieces. Have students describe the pieces. Have students label each piece $\frac{1}{2}$. Talk about the meaning of the 1 (it is 1 part) and the meaning of the number 2 (there are 2 parts in the whole strip).

Introduce the terms *whole*, *fraction*, *unit fraction*, *numerator*, and *denominator*. Add them to your mathematics word wall.

Continue with another color, asking students to fold the piece into four equal parts. Have a similar discussion about the pieces. Proceed with eighths, thirds, and sixths.

Notes

Sample PLANNING PAGE (Continued)

Questions/Prompts:

Ask questions that directly relate new vocabulary to the work students are doing.

Be sure to give students plenty of time to talk about what they noticed. Important ideas that should come out of the discussion include:

- The whole is the same size for each fraction.
- A fraction is a part of the whole.
- The smaller the denominator the larger the piece (thirds are greater than fourths).
- The numerator indicates it is one part of the whole. These are called unit fractions.
- The denominator indicates the number of equal-size pieces in the whole.

Save these fraction strips for future work with comparing fractions.

Differentiating Instruction:

Struggling Students: Watch for students who may struggle with figuring out how to fold the fractions, particularly thirds and sixths.

Students need to label each part with a unit fraction. Give struggling students the opportunity to talk about the size of unit fractions. It may help these students to cut the pieces apart after labeling them. Ask them to reconstruct the whole.

Have extra prepared strips for students who are not successful in folding the fraction strips into equal parts. It is important to let them try—several times.

Extension: Although it is not expected at this grade level, some students may want to experiment folding fractions with other denominators.

Notes

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

Number and Operations—Fractions[1]

4.NF.A*

Cluster A

[1] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Extend understanding of fraction equivalence and ordering.

STANDARD 1 **4.NF.A.1:** Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

STANDARD 2 **4.NF.A.2:** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

*Major cluster

Number and Operations—Fractions[1] 4.NF.A

Cluster A: Extend understanding of fraction equivalence and ordering.

[1] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Grade 4 Overview

Fourth graders continue to work with equivalence beginning with models and using those models to generalize a pattern and eventually a rule for finding equivalent fractions. They justify their reasoning using pictures numbers and words. In Grade 3, students compared fractions with like numerators or like denominators. They now extend that understanding to comparing fractions with different numerators and denominators reinforcing the important comparison concept that fractions must refer to the same whole.

Standards for Mathematical Practice
SFMP 2. Use quantitative reasoning.
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 5. Use appropriate tools strategically.
SFMP 7. Look for and make use of structure.
SFMP 8. Look for and express regularity in repeated reasoning.

Fourth graders extend their understanding of equivalent fractions reasoning with visual models. They look for patterns both physical (when I double the number of pieces in the whole pizza, I double the number of pieces that I ate.) and think about these patterns in terms of the meaning of the numerator and the denominator. Providing experiences with appropriate visual models will help students to develop understanding rather than just following a rule that has no meaning. Through finding and discussing patterns students construct mathematical arguments to explain their thinking as they build sets of equivalent fractions. All of this work supports the fundamental structure of fractional numbers that is critical to all future work with fractions in this domain.

Related Content Standards

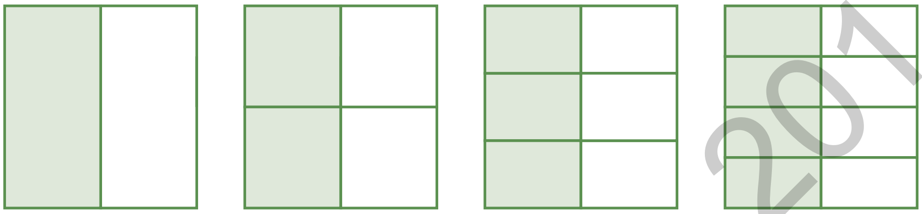
3.NF.A.2 3.NF.A.3

STANDARD 1 (4.NF.A.1)

Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Previous work in Grade 3 included exploring to find equivalent fractions using area models, fraction strips, and the number line. Although students looked for patterns, a formal algorithm for finding equivalent fractions was not developed. Fourth graders build on prior experiences, beginning with area models, to formally describe what happens to the number of pieces in the whole and the number of pieces shaded when they compare $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$ and $\frac{4}{8}$ using models, pictures, words and numbers.



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{1 \times 3}{2 \times 2}$ $\frac{3}{6}$ $\frac{1 \times 3}{2 \times 3}$ $\frac{4}{8}$ $\frac{1 \times 4}{2 \times 4}$

Students should be able to explain that when the number of pieces in the whole is doubled, the number of pieces in the count (the numerator) also doubles. This is true when multiplying by any factor.

Note that the Standards do not require students to simplify fractions although students may find fractions written in simpler form easier to understand. For example, if they recognize that $\frac{50}{100}$ is equivalent to $\frac{1}{2}$, they may choose to use $\frac{1}{2}$ since the two fractions are equivalent. Having students find equivalent fractions "in both directions" may help students to realize that fractions can be written in simpler form without formally simplifying fractions.

What the TEACHER does:

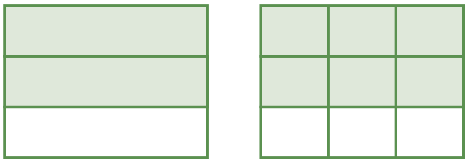
- Provide students with different models to use in building sets of equivalent fractions for visual representations and then write the fractions as numerals.
- Facilitate student discussions about patterns they see in sets of equivalent fractions.
- Expect students to use models and written numerals to generate a rule for finding equivalent fractions.
- Provide a variety of activities to help students build and recognize equivalent fractions.

What the STUDENTS do:

- Connect visual representations of equivalent fractions to numerical representations.
- Use pictures, words, and numbers to explain why fractions are equivalent.
- Generate a rule for finding equivalent fractions and follow that rule.
- Recognize equivalent fractions.

Addressing Student Misconceptions and Common Errors

Students who use addition or subtraction instead of multiplication to develop sets of equivalent fractions need additional experiences with visual representations including fraction bars, areas models, and the number line. Explanations of why one multiplies or divides to find an equivalent fraction should begin with visual representations and eventually connect to the rule/algorithm.



$\frac{2}{3}$ $\frac{2 \times 3}{3 \times 3} = \frac{6}{9}$

If I triple the number of pieces in the whole, that triples the number of pieces in my count.

STANDARD 2 (4.NF.A.2)

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Students compare two fractions with different denominators by creating equivalent fractions with a common denominator or with a common numerator. Using benchmarks such as 0, $\frac{1}{2}$, or 1 will help students to determine the relative size of fractions.

Students justify their thinking using visual representations (fraction bars, area models, and number lines), numbers, and words. It is important for students to realize that size of the wholes must be the same when comparing fractions.

Use benchmark fractions,

Compare $\frac{5}{6}$ and $\frac{7}{12}$.

think

$\frac{5}{6}$ is almost 1

So $\frac{7}{12}$ is a little more than $\frac{1}{2}\left(\frac{6}{12}\right)$

$\frac{5}{6} > \frac{7}{12}$

Use common denominators,

Compare $\frac{5}{6}$ and $\frac{3}{4}$.

think

So $\frac{5}{6} = \frac{10}{12}$ and $\frac{3}{4} = \frac{9}{12}$ and $\frac{10}{12} > \frac{9}{12}$

$\frac{5}{6} > \frac{3}{4}$

Use common numerators.

Compare $\frac{3}{6}$ and $\frac{5}{8}$.

think

$\frac{3}{6} = \frac{15}{25}$ $\frac{5}{8} = \frac{15}{24}$

Since 25ths are less than 24ths, $\frac{15}{25} < \frac{15}{24}$

So

$\frac{3}{5} < \frac{5}{8}$

Students should have opportunities to justify their thinking as well as which strategy is the most efficient to use.

What the TEACHER does:

- Provide a variety of concrete materials for students to use in comparing fractions.
    - ○ Use 0, $\frac{1}{2}$, 1 as benchmarks to compare fractions.
    - ○ Find common denominators to compare fractions.
    - ○ Find common numerators to compare fractions.

      Note: Students should determine which method makes the most sense to them, realizing that they will use different methods for different situations.
- Engage students in a variety of activities and problem solving situations in which they compare fractions and justify their reasoning using pictures, words, and numbers.

What the STUDENTS do:

- Use a variety of representations to compare fractions including concrete models, benchmarks, common denominators, and common numerators.
- Determine which method makes the most sense for a given situation and justify their thinking.
    - ○ Louisa and Linda went to the movies. Each bought a small box of popcorn. Linda ate $\frac{5}{6}$ of her popcorn and Louisa at $\frac{5}{8}$ of her popcorn. Who ate more?
    - ○ Linda ate more. Because sixths are larger than eighths, $\frac{5}{6} > \frac{5}{8}$.
    - ○ Mrs. Multiple made two pans of brownies. One pan had nuts and the other was plain. Each pan was the same size. The pan of brownies with nuts has $\frac{5}{12}$ left. The pan of plain brownies has $\frac{5}{8}$ left. Which pan has less left?

      I know that $\frac{5}{12}$ is less than $\frac{1}{2}$ (which is $\frac{6}{12}$). I know that $\frac{5}{8}$ is more than $\frac{1}{2}$ (which is $\frac{4}{8}$). Therefore the pan of brownies with nuts has less than the pan with the plain brownies because $\frac{5}{12} < \frac{5}{8}$.
    - ○ Terri has collected $\frac{2}{3}$ of the money she needs to buy her mom's birthday present. Her brother Timmy has collected $\frac{5}{6}$ of the money he needs to buy his gift. Who is closer to their goal?

      I know that $\frac{2}{3}$ is equivalent to $\frac{4}{6}$. Timmy has $\frac{5}{6}$, Terry has $\frac{4}{6}$. Timmy is closer to his goal because $\frac{5}{6} > \frac{2}{3}$ ($\frac{4}{6}$).

Addressing Student Misconceptions and Common Errors

It is important for students to use reasoning and number sense to compare fractions and justify their thinking. Students who forget that the larger the number in the denominator, the smaller the piece, may base their comparisons on incorrect notions. These students need additional practice with concrete models and making connections to the written numerals. When comparing fractions, students must consider the size of the whole. One-half of a large box of popcorn is greater than $\frac{1}{2}$ of a small box of popcorn. Take time to provide a variety of experiences for students to make sense of these important concepts.

Number and Operations—Fractions[1]
4.NF.B*

[1] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Cluster B

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

STANDARD 3

4.NF.B.3: Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:*

$$\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$$

$$2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$$

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

STANDARD 4

4.NF.B.4: Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. *For example, use a visual fraction model to represent* $\frac{5}{4}$ *as the product* $5 \times \frac{1}{4}$, *recording the conclusion by the equation* $\frac{5}{4} = 5 \times \frac{1}{4}$.

b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express* $3 \times \frac{2}{5}$ *as* $6 \times \frac{1}{5}$, *recognizing this product as* $\frac{6}{5}$. *(In general,* $n \times \frac{a}{b} = \frac{(n \times a)}{b}$.*)*

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat* $\frac{3}{8}$ *of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

*Major cluster

Number and Operations—Fractions[1] 4.NF.B

Cluster B: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

[1] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Grade 4 Overview

Fourth graders continue to develop understanding of fractions as numbers composed of unit fractions (for example, $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$). They also extend their understanding that fractions greater than 1 can be expressed as mixed numbers (for example, $\frac{12}{5} = \frac{5}{5} + \frac{5}{5} + \frac{2}{5} = 2\frac{2}{5}$). They connect their understanding of addition and subtraction of whole numbers as adding to/joining and taking apart/separating to fraction contexts using fractions with like denominators. They begin with visual representations, including area models, fraction strips, and number lines, and connect these representations to written equations.

First experiences with multiplication of a fraction by a whole number begin with connecting the meaning of multiplication of whole numbers to multiplication of a fraction by a whole number (for example, $5 \times \frac{1}{4}$ means 5 groups of $\frac{1}{4}$) using visual representations. Following many experiences modeling multiplication with unit fractions by whole numbers, students continue to work with other fractions. They solve problems by modeling using area models, fraction strips, and number lines and explain their reasoning to others.

Standards for Mathematical Practice

SFMP 1. Make sense of problems and persevere in solving them.
SFMP 2. Use quantitative reasoning.
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 5. Use appropriate tools strategically.
SFMP 6. Attend to precision.
SFMP 7. Look for and make use of structure.
SFMP 8. Look for and express regularity in repeated reasoning.

Students extend their work with unit fractions to composing and decomposing non-unit fractions. In doing so, they reason about fractions as numbers (quantitatively) and understand that fractions, like whole numbers, represent a "count" of something. The main difference is the "something" includes part of a whole. Problem solving contexts reinforce the meaning of addition and subtraction, presenting opportunities for students to relate previous work with addition and subtraction situations with whole numbers to adding and subtracting fractions. They use models including area models, fraction strips, and number lines, and connect those visual models to written equations when they are ready. They build on previous understandings of the meaning of the numerator and denominator (precision) to see the structure of addition and subtraction and explain what is happening when they add and subtract fractions (for example, why they add or subtract numerators but keep the same denominator).

Related Content Standards

1.OA.A.1 2.OA.A.1 3.NF.A.2 3.G.A.2 5.NF.A.1 5.NF.A.2

Notes

STANDARD 3 (4.NF.B.3)

Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.

Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Unit fractions are fractions with a numerator of 1. Third graders' experiences with fractions focused on unit fractions. Their work with non-unit fractions was limited to using visual models such as fraction strips and number lines to see that fractions such as $\frac{3}{4}$ are composed of three jumps of $\frac{1}{4}$ on the number line. This is an important concept as students prepare to add and subtract fractions. Fourth grade experiences extend to composing and decomposing fractions greater than 1 (improper fractions) and mixed numbers into unit fractions Students use prior knowledge of using concrete fraction representations for whole numbers to move between mixed numbers and fractions.

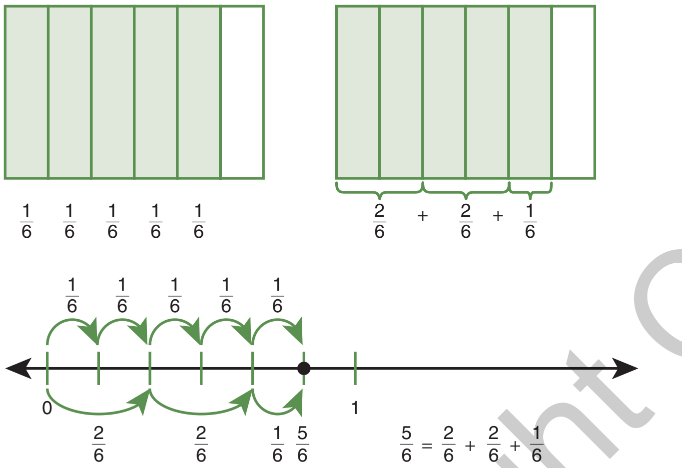
What the TEACHER does:

- Provide a variety of experiences for students to compose and decompose fractions, including fractions greater than 1 and mixed numbers, into unit fractions using concrete and pictorial representations, words, and numbers.

What the STUDENTS do:

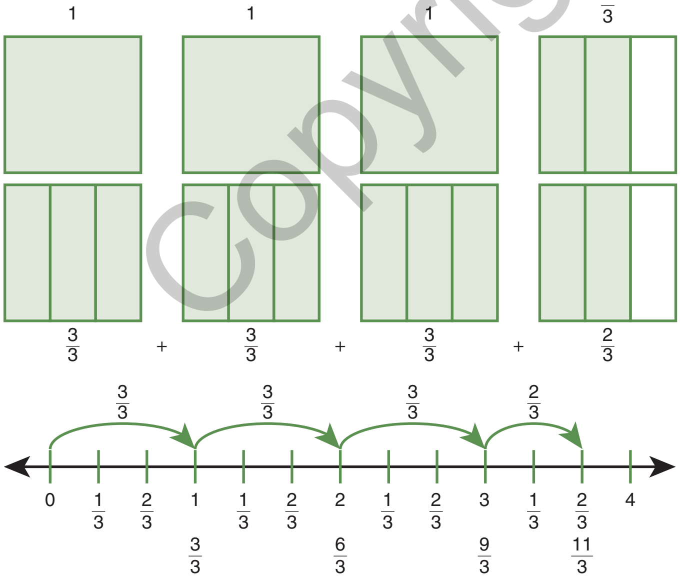
- Compose and decompose fractions, including fractions greater than 1 and mixed numbers, into unit fractions using concrete and pictorial representations including the number line.
- Explain their reasoning using pictures, words, and numbers.

Representations for $\frac{5}{6}$



$\frac{5}{6} = \frac{2}{6} + \frac{2}{6} + \frac{1}{6}$

Representations for $3\frac{2}{3}$



$3\frac{2}{3} = \frac{9}{3} + \frac{2}{3} + \frac{11}{3}$

Addressing Student Misconceptions and Common Errors

Although students may be able to decompose a fraction into unit fractions (that is, $\frac{4}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$), when given the unit fractions to compose into a fraction, they may think they need to add denominators as well as numerators. This misconception can be avoided by giving students multiple opportunities with various concrete models, pictures, and the number line and making explicit connections to written equations.

a. *Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.*

This Standard begins with an understanding that addition and subtraction of fractions has the same meaning as addition and subtraction of whole numbers, although the process of adding and subtracting is different with fractions. Remember expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. Addition and subtraction work is limited to examples with like denominators.

What the TEACHER does:

- Give students activities that relate the meaning of addition and subtraction of fractions to addition and subtraction of whole numbers.
- Use problem solving situations with addition and subtraction of fractions relating to the same whole, and have the students determine which operation should be used to solve the problem. (See Table 1, page 254.)

What the STUDENTS do:

- Use a variety of materials to model and describe various problem situations that require adding and subtracting fractions.

Addressing Student Misconceptions and Common Errors

Students need not actually add or subtract fractions at this point, although many of them will be ready. Students who struggle with identifying a situation as an addition situation or a subtraction situation need more experience solving problems that require addition or subtraction. Modeling such situations using fraction pieces will help them to relate these operations to previous work with whole numbers (Table 1, page 254).

Notes

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$

$2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$

This Standard includes work with improper fractions and mixed numbers.

What the TEACHER does:

- Provide a variety of activities in which students must decompose a fraction into fractions with the same denominator. Use a variety of denominators.
  - Begin with decomposing a fraction into unit fractions.

    $$\frac{5}{12} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$$

  - Ask students to combine the unit fractions to show other addends that compose the fraction.

    $$\frac{5}{12} = \frac{1}{12} + \frac{1}{12} + \frac{3}{12}$$

    $$\frac{5}{12} = \frac{2}{12} + \frac{3}{12}$$

- Facilitate discussions in which students use visual models, including area models and the number line, to justify their thinking.
- As students demonstrate understanding with fractions less than one, extend to activities with fractions greater than 1 and mixed numbers.

$$\frac{5}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

$$\frac{5}{4} = \frac{2}{4} + \frac{3}{4}$$

$$\frac{5}{4} = \frac{4}{4} + \frac{1}{4}$$

$$2\frac{3}{8} = \frac{8}{8} + \frac{8}{8} + \frac{3}{8}$$

$$2\frac{3}{8} = \frac{16}{8} + \frac{3}{8}$$

- Encourage students to find many different ways to decompose fractions and explain their reasoning.

What the STUDENTS do:

- Decompose fractions less than 1 into fractional parts with the same denominator using models, pictures, words, and numbers.
- Explain their reasoning using visual models.
- Decompose fractions greater than 1 into fractional parts with the same denominator using models, pictures, words, and numbers.
- Explain their reasoning using visual models and equations.
- Decompose mixed numbers into fractional parts with the same denominator using models, pictures, words, and numbers.
- Explain their reasoning using visual models and equations.

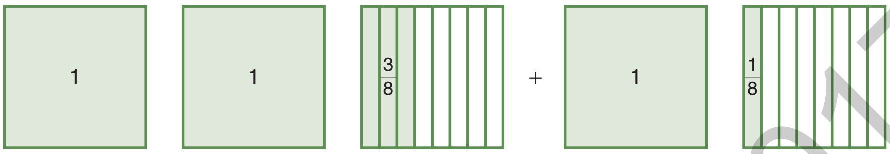
Addressing Student Misconceptions and Common Errors

Although this work may seem obvious to some students, it is important to take the time to build this concept because it lays the foundation for adding and subtracting fractions. Students who see fractions as composed of smaller parts develop the understanding that when they add or subtract fractions, the numerator describes the count of pieces and the denominator describes the piece. Carefully developing this concept now will avoid misconceptions many students have when adding two fractions with unlike denominators.

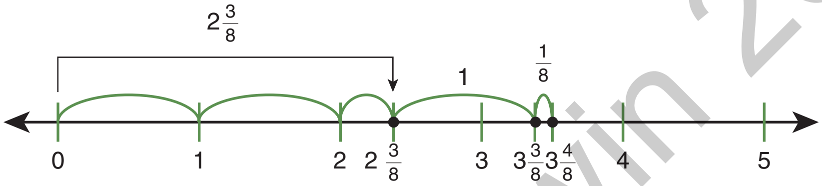
c. *Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.*

After students have had ample experience composing and decomposing various fractions and mixed numbers, they work with adding and subtracting fractions with like denominators. This Standard includes work with fractions less than one, fractions greater than one, and mixed numbers. At this point students do not need to regroup or decompose mixed numbers. When adding and subtracting mixed numbers, students should use concrete materials and develop strategies that make sense to them.

$2\frac{3}{8} + 1\frac{1}{8} =$



$1 + 1 + 1 + \frac{3}{8} + \frac{1}{8} = 3\frac{4}{8}$



$2\frac{3}{8} + 1\frac{1}{8} =$

$2 + \frac{3}{8} + 1 + \frac{1}{8} = 3\frac{4}{8}$

Note that this Standard and 4.NF.B.3.d should be taught simultaneously so that students have contexts in which to build understanding and determine whether their answers make sense.

d. *Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.*

(Refer to Table 1, page 254.) Students have experienced all of these situations using whole numbers in earlier grades. Use similar situations that involve fractions and mixed numbers with like denominators for students to solve as they continue to add and subtract fractions using pictures, words, and numbers. Be sure to give students many opportunities to consider the reasonableness of their answers.

What the TEACHER does:

- Provide students with fraction models, including area models, fraction bars, and number lines, to use as they solve addition and subtraction of fraction problems.
- Scaffold examples and problems.
    - Adding and subtracting fractions less than 1.
    - Adding and subtracting fractions greater than 1.
    - Adding and subtracting mixed numbers (with no regrouping).
- Expect students to solve problems using visual representations and provide opportunities to have them make explicit connections to numerical representations.
- Facilitate discussions in which students explain their thinking using materials, pictures, words, and numbers.

What the STUDENTS do:

- Use concrete materials and pictures to solve a variety of problems involving addition and subtraction of fractions and mixed numbers.
- Connect visual models to addition and subtraction equations.
- Explain their thinking using models, pictures, numbers, and words.

Addressing Student Misconceptions and Common Errors

Watch for students who may add or subtract denominators when adding and subtracting fractions. These students need additional concrete experiences and specific questions about whether their answer is reasonable. For example, if a student adds $\frac{2}{3} + \frac{3}{3}$ and gets a sum of $\frac{5}{6}$, talk about the value of the addends and the value of the sum to realize that the answer should be greater than 1.

Number lines and visual models will also reinforce correct thinking. It is important that students understand that the numerator tells the count (how many pieces) and the denominator describes the piece. Since the pieces are the same size, the numerator (count) is added and the description of the pieces does not change. When I add 2 pieces that are thirds to 3 pieces that are thirds I will get 5 pieces that are thirds.

Notes

STANDARD 4 (4.NF.B.4)

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

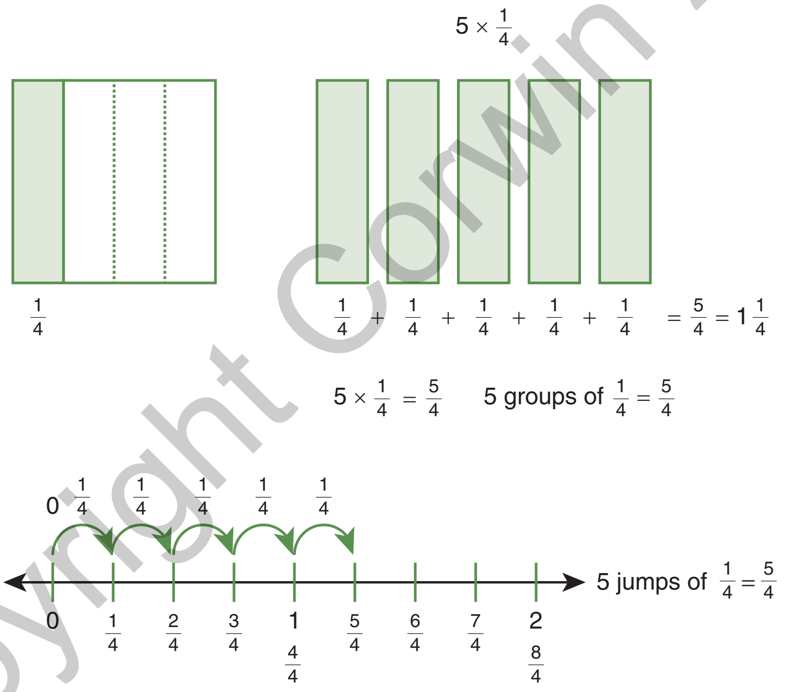
Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Students need a variety of experiences to understand that the meaning of multiplication with fractions is the same as the meaning of multiplication with whole numbers. They begin by thinking about a whole number of fractional pieces or the number of groups of a given fraction. Note that Standard 4.NF.B.4.d should be taught at the same time as Standards 4.NF.B.4.a and 4.NF.B.4.b, using appropriate numbers so that students have contexts in which to build understanding rather than focusing only on the numbers.

a. *Understand a fraction* $\frac{a}{b}$ *as a multiple of* $\frac{1}{b}$. For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times \frac{1}{4}$, recording the conclusion by the equation $\frac{5}{4} = 5 \times \frac{1}{4}$.

This Standard builds on experiences with decomposing fractions into unit fractions and connecting that understanding to multiplication.

$\frac{5}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ or 5 groups of $\frac{1}{4}$, which can be represented as $5 \times \frac{1}{4}$.



At this point, some students may find a pattern and a more efficient algorithm (procedure) for multiplying a whole number times a fraction (that is, multiply the whole number times the numerator of the fraction) but it is not an expectation for all students. The critical focus of this Standard is to develop an understanding of what is happening when multiplying a whole number times a unit fraction by relating the process to the meaning of multiplication.

Note: The language of this Standard can be confusing. When we multiply a fraction by a whole number we are thinking a whole number of groups of a given fraction (for example, $5 \times \frac{1}{4}$). Students will multiply whole numbers by fractions in Grade 5 (for example, $\frac{1}{4} \times 5$).

What the TEACHER does:

- Review the meaning of multiplication of whole numbers as one factor representing the number of "groups" and the other factor representing the number of items in a group using physical representations and the number line.
  - ○ 3 × 4 means I have 3 groups of 4.
  - ○ 3 × 4 means 3 jumps of 4 on the number line.
- Extend this meaning to physical representations of a unit fraction multiplied by a whole number using problem solving contexts.

  I bought 3 boxes of crackers. Each box had $\frac{1}{4}$ lb. What is the total weight of the crackers?
  - ○ $3 \times \frac{1}{4}$ means I have three groups of $\frac{1}{4}$.
  - ○ $3 \times \frac{1}{4}$ means 3 jumps of $\frac{1}{4}$ on the number line.
- Provide students with many experiences to model a whole number times a unit fraction.
- Facilitate student discussions in which students explain their thinking using pictures, words, and numbers.
- Watch for students who see a pattern and may generalize a "rule" for multiplication. Be certain they understand why their rule works.

What the STUDENTS do:

- Model and explain the meaning of whole number multiplication.
- Extend the model to examples in which they multiply a fraction by a whole number.
- Explain their thinking using pictures, words, and numbers.

Addressing Student Misconceptions and Common Errors

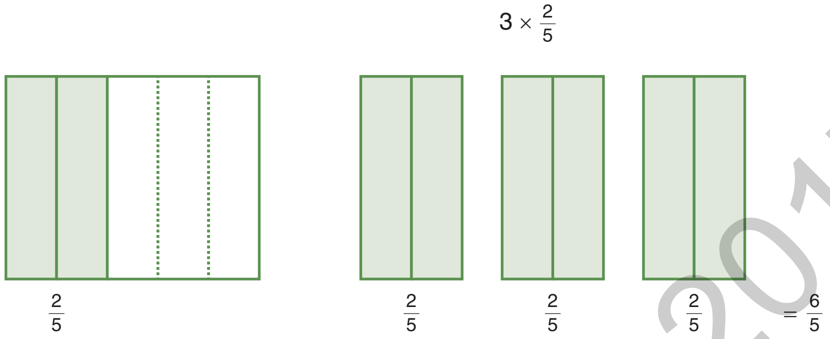
Students may see "the rule" without really understanding the connection to the meaning of multiplication. It is especially important to expect these students to model and explain their thinking rather than simply using the rule.

Some students may want to put a denominator on the whole number to relate this work to previous work with addition and subtraction of fractions. These students need additional opportunities to solve problems that provide a context for the meaning of multiplication as it relates to fractions. Once they can model the situation, help them connect the model to a written equation. Ask questions about what is happening and give them opportunities to explain what they are doing.

Notes

b. *Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number.* For example, use a visual fraction model to express $3 \times \frac{2}{5}$ as $6 \times \frac{1}{5}$, recognizing this product as $\frac{6}{5}$. (In general, $n \times \frac{a}{b} = \frac{(n \times a)}{b}$.)

Once students can multiply a whole number times a unit fraction, they extend that understanding to multiplying a whole number times any fraction first with visual models and then connecting those models to numerical representations.



Give enough practice with models and connecting those models to written equations to help students see the structure of multiplication working with fractions. The whole number represents the number of groups and the fraction represents the number of items in each group. Because the numerator gives the count of how many pieces and the denominator describes the pieces, multiplying the number of groups times the count of items in each group (the numerator) will tell the total number of pieces. Because the denominator describes the piece it does not change.

What the TEACHER does:

- Provide a variety of problem contexts for students to model multiplication of any fraction by a whole number.
- Ask questions to facilitate student explanations of their reasoning.
    - How many groups do you have?
    - How many are in each group?
    - What would this look like if you model it with fraction pieces or on the number line?
- Scaffold to problems that include fractions greater than 1 and mixed numbers.
- Help students make explicit connections between models and written equations.
- Watch for students who are able to generalize a rule for multiplying a whole number times any fraction to be certain they understand why it works as well as how it works.

What the STUDENTS do:

- Solve a variety of problems involving multiplication of a fraction by a whole number using models, including area models, fraction strips, and number lines.
- Explain their reasoning using pictures, words, and numbers.

Addressing Student Misconceptions and Common Errors

Watch for students who are rewriting the whole number as a name for 1 (for example, writing 4 as $\frac{4}{4}$ rather than $\frac{4}{1}$). In these situations students should be thinking of the whole number as the number of groups and therefore they do not need to rewrite it as a fraction.

c. *Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.* For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

This Standard should be taught as the same time as parts a and b. Students should have a variety of contexts in which to solve multiplication of whole number and fraction examples so they can estimate, model, and determine whether their answers are reasonable.

What the TEACHER does:

- Provide students with a variety of multiplication problem situations (Table 2, page 256) using multiplication of fractions and mixed numbers by a whole number as contexts.
- Scaffold student experiences.
    - ○ Begin with a fraction times a whole number. $3 \times \frac{4}{5}$
    - ○ Multiply a fraction greater than 1 by a whole number. $3 \times \frac{14}{4}$
    - ○ Multiply a mixed number by a whole number. $9 \times 1\frac{7}{10}$
- Expect students to model and explain their solutions using concrete and pictorial representations, words, and numbers.

What the STUDENTS do:

- Use models to solve a variety of problem situations involving multiplying a whole number times a fraction or mixed number.
- Explain their reasoning using models, pictures, words, and numbers.
- Talk about any patterns they see when multiplying a fraction or mixed number times a whole number in relation to the meaning of the whole number as the number of groups, the numerator and denominator of the fraction, and the meaning of multiplication.

Addressing Student Misconceptions and Common Errors

Students who struggle with identifying and modeling multiplication situations from Table 2 (page 256) need more experience with these situations and using appropriate models. Use fractions of reasonable size so that students can focus both on the situation and why it is a multiplication situation as well as deal with the numbers they need to use to solve the problem. Do not teach students to look for key words (such as *of*) because this does not support making sense of the situation and what is happening with the fractions.

Notes