

CHAPTER 3

LAYING YOUR FOUNDATION

It Starts With Big Ideas, Essential Questions, and Standards

As required by a new district policy, two veteran third-grade teachers, Rosemarie and Mike, sat down with their school administrative leader to review their students' benchmark assessments. Rosemarie, who had not yet seen the results, had been nervous all day about this meeting. She knew that Linda, the school principal, supported their work.

Linda pulled up the screen with the results and displayed them. "Let's just take a few minutes to look at the data before we discuss. Let's start with the successes. I am noticing that the students performed beautifully on geometry concepts. These scores are way up from last year."

Rosemarie commented, "We really hit geometry hard this year. In fact, I was truly amazed with their understanding."

Linda said, "I am so glad that all this effort paid off! Now, let's look at what we need to work on."

Rosemarie said, "My students were completely confused about the representations used for fractions."

Mike exclaimed, "Mine were, too! Do you think it has anything to do with the new standards? We always taught

fractions, but we never used those representations that were on the test."

Linda replied, "I think you are on to something, Mike. How could we strategically plan for the new standards so that we can create the same kind of success you had with geometry?"

Rosemarie and Mike's surprise about the assessment results may mirror the feelings of many teachers after states and districts implement new standards. In this chapter, we will focus on big ideas, essential questions, and standards as the building blocks of a lesson taught at the 3–5 grade levels. We will also address the following questions:

- What are state standards for mathematics?
- What are essential questions?
- What are process standards?

WHAT ARE STATE STANDARDS FOR MATHEMATICS?

Grade-level standards describe what students should know and be able to do at the end of the grade level. For many years, research studies of mathematics education concluded that in order to improve mathematics achievement in the United States, standards needed to become more focused and coherent. The development of common mathematics **standards** began with research-affirmed **learning progressions** highlighting what is known about how students develop mathematical knowledge, skills, and understanding. The resulting document became known as the *Common Core State Standards for Mathematics (CCSS-M)* (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The landmark document was intended to be a set of shared goals and expectations for the knowledge and skills students need in mathematics at each grade level. The overall goal was college and career readiness.

Currently, the majority of states have adopted the *Common Core State Standards for Mathematics* as their own state standards. However, it is important to note that while many states adopted the CCSS-M, others have updated, clarified, or otherwise modified them, adopting the updated set as their new state standards. A few states have written their own standards.

Most standards documents are composed of **content standards** and **process standards** of some kind. It is important to recognize that no state standards describe or recommend what works for all students. Classroom teachers, not the standards, are the key to improving student learning in mathematics. The success of standards depends on teachers knowing how to expertly implement them. It is important as a teacher to be very knowledgeable about your own state standards and what they mean—not only at your grade level but also at the one above and below the one you teach. They are at the heart of planning lessons that are engaging, purposeful, coherent, and rigorous.

Regardless of whether your state has adopted CCSS-M, has modified the standards, or has written its own, the **big ideas** of 3–5 mathematics are universal. Big ideas are statements that describe concepts that transcend grade levels. Big ideas provide focus on specific content. Here are the big ideas for Grades 3 through 5 on fractions.

Third Grade

In third grade, students use visual models, including area models, fraction strips, and the number line to develop conceptual understanding of the meaning of a fraction as a number in relationship to a defined whole. They work with unit fractions to understand the meaning of the numerator and denominator. Students build equivalent fractions and use a variety of strategies to compare fractions. In Grade 3, fractions are limited to halves, thirds, fourths, sixths, and eighths.

Fourth Grade

Fourth graders extend understanding from the third-grade experiences, composing fractions from unit fractions and decomposing fractions into unit fractions. They then apply this understanding to addition and subtraction with like denominators. Students begin with visual models and progress to making generalizations for addition and subtraction of fractions with like denominators. They compare fractions from the same whole using a variety of strategies. Students build equivalent fractions with denominators of 10 and 100 and connect that work to decimal notation for tenths and hundredths. Students add and subtract fractions with like denominators.

Fifth Grade

Fifth-grade students build on previous experiences with fractions and use a variety of visual representations and strategies to add and subtract fractions with unlike denominators. Problem solving provides contexts for students to use mathematical reasoning to determine whether answers make sense. They extend their interpretation of a fraction as part of a whole to fractions as a division representation of the numerator divided by the denominator. Students continue to build conceptual understanding of multiplication of fractions using visual models and problem-solving contexts. Once conceptual understanding is established, students generalize efficient procedures for multiplying and dividing fractions.



While the major topics in third- through fifth-grade mathematics are the same, states may vary in their focus at each grade level. Compare your state standards for your grade level with the summary above. Are there any differences? If so, what are they? Briefly describe them here.

WHAT ARE ESSENTIAL QUESTIONS?

It is estimated that over the course of a career, a teacher can ask more than two million questions (Vogler, 2008). If teachers are already asking so many questions, why do we need to consider essential questions? An **essential question** is a building block for designing a good lesson. It is the thread that unifies all of the lessons on a given topic to bring the coherence and purpose discussed previously. Essential questions are purposefully linked to the big idea to frame student inquiry, promote critical thinking, and assist in transferring learning. (See Chapter 5 for more information on essential questions in transfer lessons.) As a teacher, you will revisit your essential question(s) throughout your unit.

Essential questions include some of these characteristics:

- *Open-ended.* These questions usually have multiple acceptable responses.
- *Engaging.* These questions ignite lively discussion and debate and may raise additional questions.
- *High cognitive demand.* These questions require students to infer, evaluate, defend, justify, and/or predict.
- *Recurring.* These questions are revisited throughout the unit, school year, other disciplines, and/or a person's lifetime.
- *Foundational.* These questions can serve as the heart of the content. Students need to understand foundational questions in order to understand the content that follows.

Not all essential questions need to have all of the characteristics. Here are some examples of essential questions that follow from big ideas for 3–5.

- When and why do people estimate?
- How are fractions used in real life?
- How do we know that fractions are numbers?
- How are common and decimal fractions alike and different?
- How do I identify the whole when working with fractions?
- How does changing the size of a whole affect the size or amount of a fractional part?
- What patterns do you see when we look at place value?
- What would life be like if there were no numbers?
- What do mathematicians do when they get stuck on a problem?



Look at the list of sample essential questions. Decide which characteristics describe which question. Note any thoughts or comments below.

WHAT ARE PROCESS STANDARDS?

Up to this point, we have been discussing content standards. However, every state also has a set of standards that define the **habits of mind** students should develop through mathematics. In 1989, the National Council of Teachers of Mathematics (NCTM) introduced these standards as process standards, stating that “what we teach [in mathematics] is as important as how we teach it” (NCTM, 1991), encouraging us to teach mathematics through these processes. Those standards are the following:

- 1. Problem solving:** Students use a repertoire of skills and strategies for solving a variety of problems. They recognize and create problems from real-world situations within and outside mathematics to find solutions.
- 2. Communication:** Students use mathematical language, including terminology and symbols, to express ideas precisely. Students represent, discuss, read, write, and listen to mathematics.
- 3. Reasoning and proof:** Students apply inductive and deductive reasoning skills to make, test, and evaluate statements to justify steps in mathematical procedures. Students use logical reasoning to analyze and determine valid conclusions.
- 4. Connections:** Students relate concepts and procedures from different topics in mathematics to one another and make connections between topics in mathematics and other disciplines.
- 5. Representations:** Students use a variety of representations, including graphical, numerical, algebraic, verbal, and physical, to represent, describe, and generalize. They recognize representation as both a process and a product.

The Common Core State Standards have eight **Standards for Mathematical Practice** (SMPs), which also describe the habits of mind students should develop as they do mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The SMPs listed below are the same across all grade levels.

- 1. Make sense of problems and persevere in solving them.** Students work to understand the information given in a problem and the question that is asked. They use a strategy to find a solution and check to make sure their answer makes sense. As third-, fourth-, and fifth-grade students work to make sense of multiplication and division, problem solving helps them develop conceptual understanding. This is also true with the focus on fractions in these grades.
- 2. Reason abstractly and quantitatively.** Students make sense of quantities and their relationships in problem situations. They develop operation sense by associating context to numbers. Reasoning in problem situations using concrete materials helps students understand the meaning of multiplication and division and build a foundation for work with fractions.
- 3. Construct viable arguments and critique the reasoning of others.** Students in Grades 3 through 5 explain their thinking, justify, and communicate their conclusions both orally and in writing. Mathematical discussions should be a common expectation in mathematics lessons. Explaining thinking and listening to each other’s thinking helps develop deeper conceptual understanding.
- 4. Model with mathematics.** Students use representations, models, and symbols to connect conceptual understanding to skills and applications. They may also represent or connect what they are learning to real-world problems.
- 5. Use appropriate tools strategically.** Students in Grades 3 through 5 use a variety of concrete materials and tools, such as counters, tiles, cubes, rubber bands, and physical number lines, to represent their

thinking when solving problems. Representations such as making equal groups, arrays, and area models help students make connections between multiplication and division as well as the importance of place value in understanding these operations. Bar models, area models, and the number line help students understand fraction concepts.

6. **Attend to precision.** Students learn to communicate precisely with each other and explain their thinking using appropriate mathematical vocabulary. Students in Grades 3 through 5 expand their knowledge of mathematical symbols, which should explicitly connect to vocabulary development.
7. **Look for and make use of structure.** Students discover patterns and structure in their mathematics work. For example, students begin their work using unit fractions, which helps them learn the meaning of numerator and denominator. They extend understanding of unit fractions to other common fractions as they develop a sense of equivalence, addition, and subtraction of all fractions, including mixed numbers. The relationship between multiplication and division of whole numbers extends to work with fractions.
8. **Look for and express regularity in repeated reasoning.** Learners notice repeated calculations and begin to make generalizations. By recognizing what happens when multiplying tens or hundreds, students extend that understanding to more difficult problems. This standard mentions shortcuts. However, shortcuts are only appropriate when students discover them by making generalizations on their own and understand why they work.

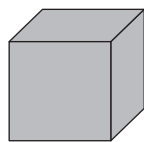
The SMPs are not intended to be taught in isolation. Instead, you should integrate them into daily lessons because they are fundamental to thinking and developing mathematical understanding. As you plan lessons, determine how students use the practices in learning and doing mathematics.

Both sets of standards overlap in the habits of mind that mathematics educators need to develop in their students. These processes describe practices that are important in mathematics. Not every practice is evident in every lesson. Some lessons/topics lend themselves to certain practices better than others. For instance, you might use **classroom discourse** to teach geometry.

Example: Manny

Manny, a fourth-grade teacher, engages his students in geometry to construct viable arguments and critique the reasoning of others.

Manny: What is this shape?



Billy: It is a box.

Manny: Why do you call it a box?

Billy: Because you can put things in it.

Mario: Yes, but we learned that box is not a math word. It is a square.

Manny: Why do you think it is a square?

Mario: Because it has four angles (*as Mario points to the four angles on the front face of the shape*).

Manny: Does everyone agree with Mario's reasoning?

Rosa: I don't. I think it is a cube. Mario said it has four angles but that is just the front flat square. (*Rosa places her hand over the front face to make her point.*) This has other faces. It is three-dimensional and squares are only two dimensions.



Building Unit Coherence

One of the best ways to build coherence between and among lessons within your unit is through the big ideas, essential questions, and standards. Keep in mind that connecting individual lessons through these three main elements promotes in-depth conceptual understanding, supports coherence, and unifies individual lessons. A big part of creating a coherent unit is strategically deciding how these three elements will be connected. Consider mapping the three components for the entire unit as you develop the lesson plan (Figure 3.1).

Figure 3.1

Unit-Planning Template

Unit Topic:

Unit Standards	Unit Big Ideas	Unit Essential Questions



Download the Unit-Planning Template from resources.corwin.com/mathlessonplanning/3-5

**Third-Grade Snapshot****Big Ideas, Essential Questions, and Standards**

The third-grade team, Saida, Julian, and Kimi, are beginning to write their lessons on comparing fractions. After discussing the ups and downs of last year's teaching, they decided to write an essential question to avoid the misconception that when fractions use different numbers in the numerator and denominator, the fractions can represent a different amount. They decide to focus the lessons on the big idea that fractions with different numerators or denominators can represent the same part of a whole.

Big Idea(s):

Fractions with different numerators or different denominators can represent the same part of a whole.

Essential Question(s):

How do we know when two fractions are equivalent?

Content Standard(s):

Compare two fractions with the same numerator or denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole.

Mathematical Practice or Process Standards:

Construct viable arguments and critique the reasoning of others.
Make sense of problems and persevere in solving them.

See the complete lesson plan in Appendix A on page 186.



What kinds of essential questions can you ask that encompass big ideas in your class? Record some of your responses below.



Fourth-Grade Snapshot

Big Ideas, Essential Questions, and Standards

Fourth-grade teachers Adrienne and Davante are developing a unit on comparing fractions. Adrienne notices that in third grade, their students learned how to compare fractions with same numerators or same denominators. This year, the standard highlights comparing fractions that have different numerators and denominators. Davante suggests building on last year's standard with an essential question that ties both years' standards together. They decide to focus on the essential question: How do we compare fractions?

Big Idea(s):

Fractions with different numerators and denominators can represent the same part of a whole.

Essential Question(s):

How do we know when two fractions are equivalent?

Content Standard(s):

Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Mathematical Practice or Process Standards:

Model with mathematics.
Look and make use of structure.

See the complete lesson plan in Appendix A on page 191.



What kinds of essential questions can you ask that encompass big ideas in your class? Record some of your responses below.



Fifth-Grade Snapshot

Big Ideas, Essential Questions, and Standards

Boton, Chelsea, and Rodrigo discuss the essential question for the multiplication of fractions standard. Rodrigo states, "I think the big idea behind multiplication of fractions could be incorporated into the essential question. What do you both think?" Boton adds, "I like the idea of asking an open-ended question for our essential question." Chelsea offers, "How about, 'What does it mean to multiply fractions?'"

<p>Big Idea(s):</p> <p>Multiplication with fractions is similar to multiplication with whole numbers. Students grapple with similarities and differences between multiplication of whole numbers and fractions.</p>	<p>Essential Question(s):</p> <p>What does it mean to multiply fractions?</p>
<p>Content Standard(s):</p> <p>Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</p> <p>a. Interpret the product $(\frac{a}{b}) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(\frac{2}{3}) \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $(\frac{2}{3}) \times (\frac{4}{5}) = \frac{8}{15}$. (In general, $(\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}$.)</p> <p>b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</p>	<p>Mathematical Practice or Process Standard(s):</p> <p>Students make sense of problems and persevere while solving them.</p> <p>Students construct viable arguments and critique the reasoning of others.</p>

See the complete lesson plan in Appendix A on page 195.



Are there other topics in your grade level that could be guided by an essential question? Give some examples below.

Standards

LI and SC

Purpose

Tasks

Materials

Student Thinking

Lesson Structures

Form. Assess.

Lesson Launch

Lesson Facilitation

Closure



Under Construction

Now it is your turn! You need to decide what big idea, essential question, and standards you want to build a lesson around. Start with your big idea and then identify the remaining elements.

Big Idea(s):	Essential Question(s):
Content Standard(s):	Mathematical Practice or Process Standards:



Download the full Lesson-Planning Template from resources.corwin.com/mathlessonplanning/3-5
Remember that you can use the online version of the lesson plan template to begin compiling each section into the full template as your lesson plan grows.